Fermilab

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Single Bunch Instabilities at the Fermilab Recycler Ring

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FRIB/MSU Accelerator Physics and Engineering Seminars (APES)

U.S. DEPARTMENT of ENERGY
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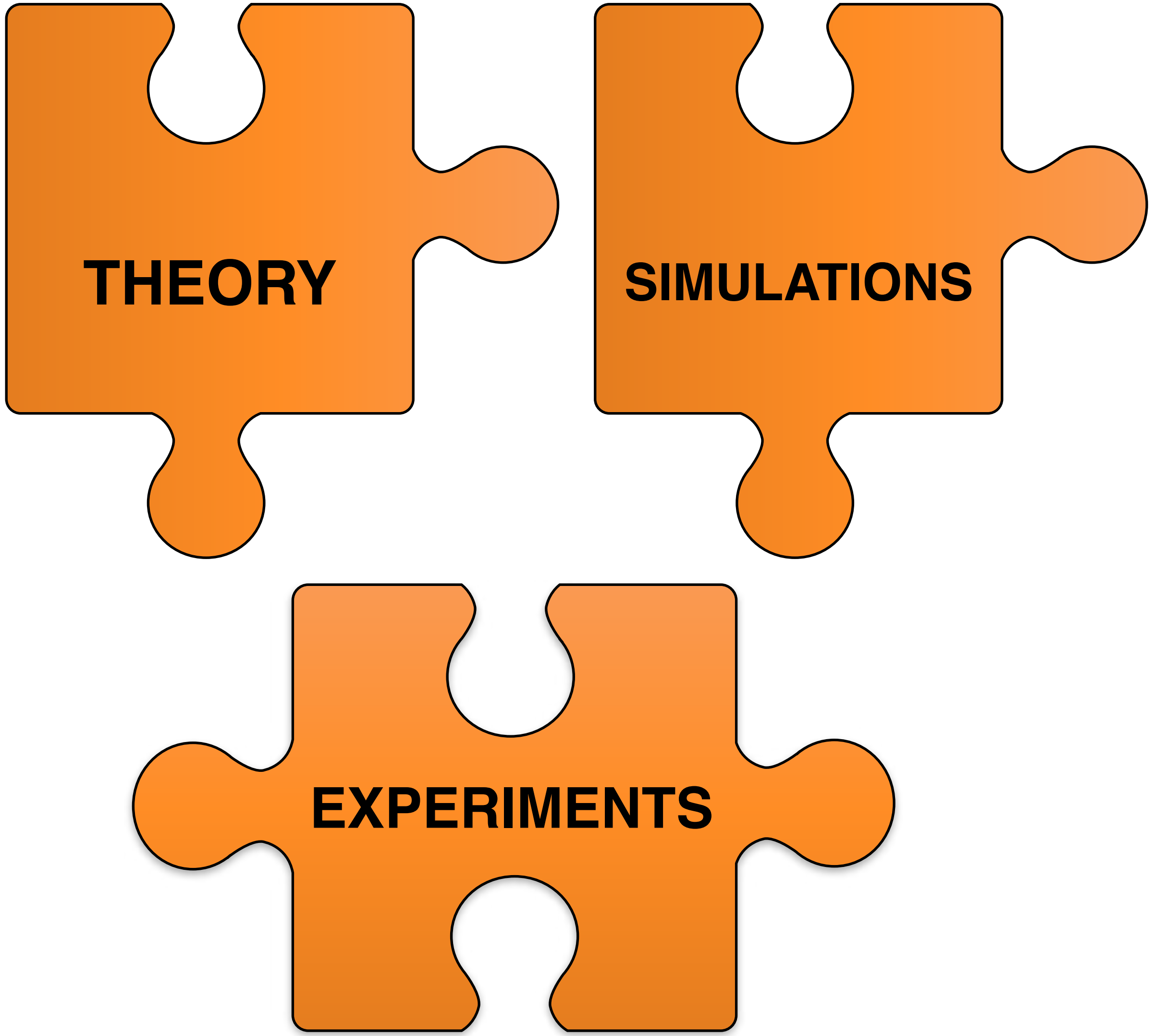
Outline

- ✓ Introduction
- ✓ Theory & Simulations (Part 1)
- ✓ Experiments
- ✓ Theory & Simulations (Part 2)
- ✓ Theory & Simulations (Part 3)
- ✓ Conclusions and Future Work



01

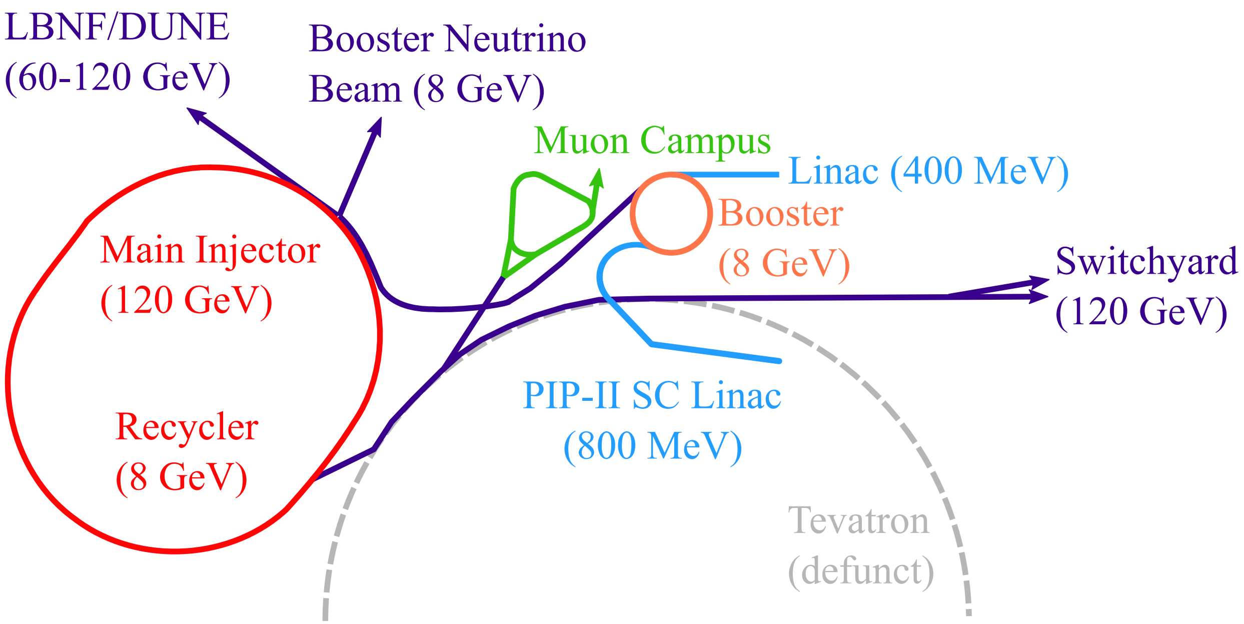
Introduction

Motivation & Introduction

As intensity is increased in the Fermilab Accelerator Complex (PIP-II and beyond), we need to better understand collective instabilities, which pose a significant limitation to achieving high-intensity beam goals.

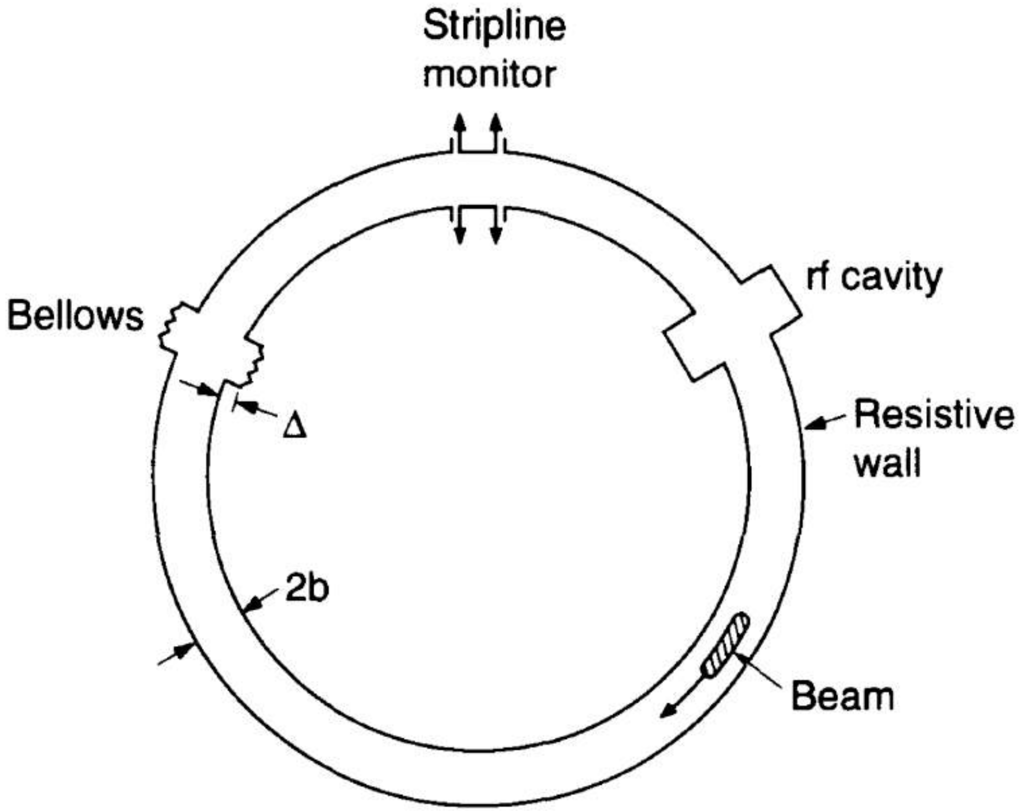
What is the maximum intensity in a single bunch we can store in the **Recycler Ring**?

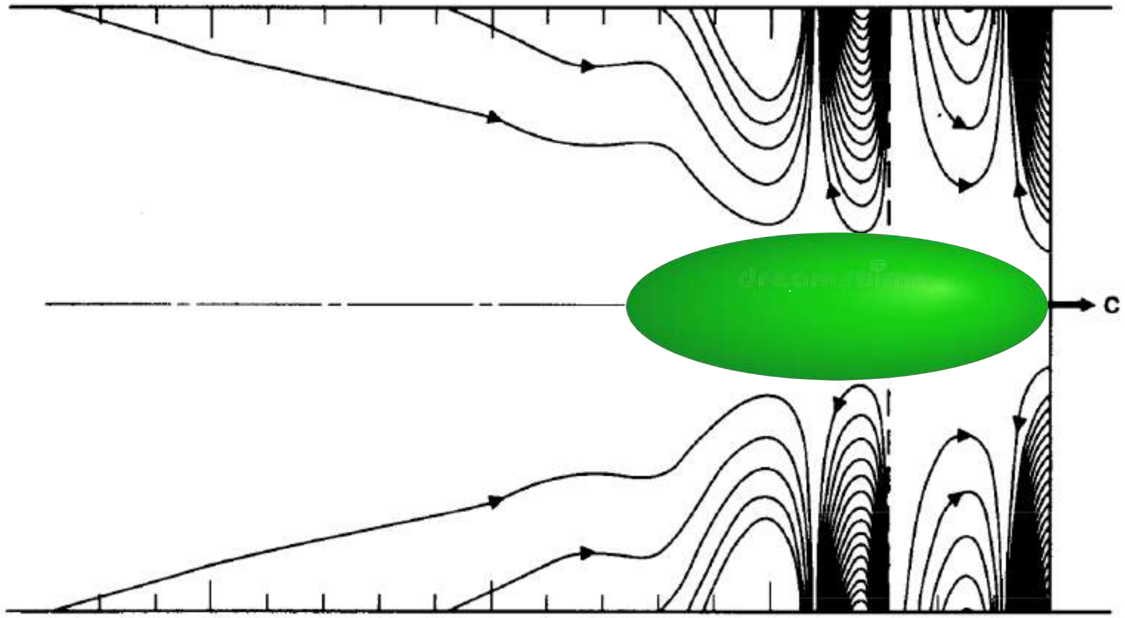
- Let's focus on a single bunch, no multi-bunch effects
- What are the main sources of impedance? What is the role of space charge?



Motivation & Introduction: Single Bunch Instabilities

Bunch will interact with **impedance elements** in the Recycler Ring and create wake fields inside the beam pipe. These wake fields will talk back to the bunch and can create instabilities over many turns.



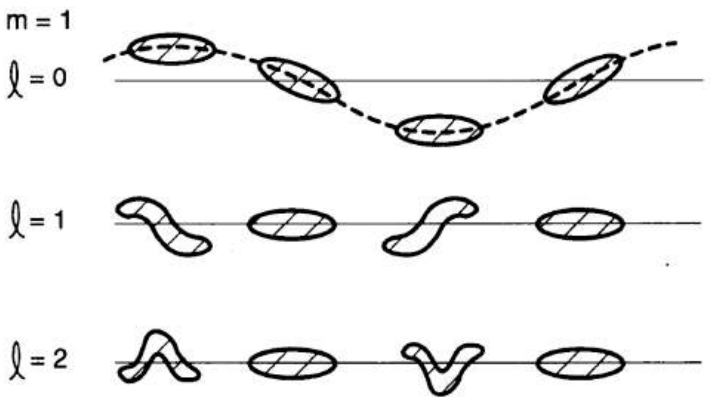


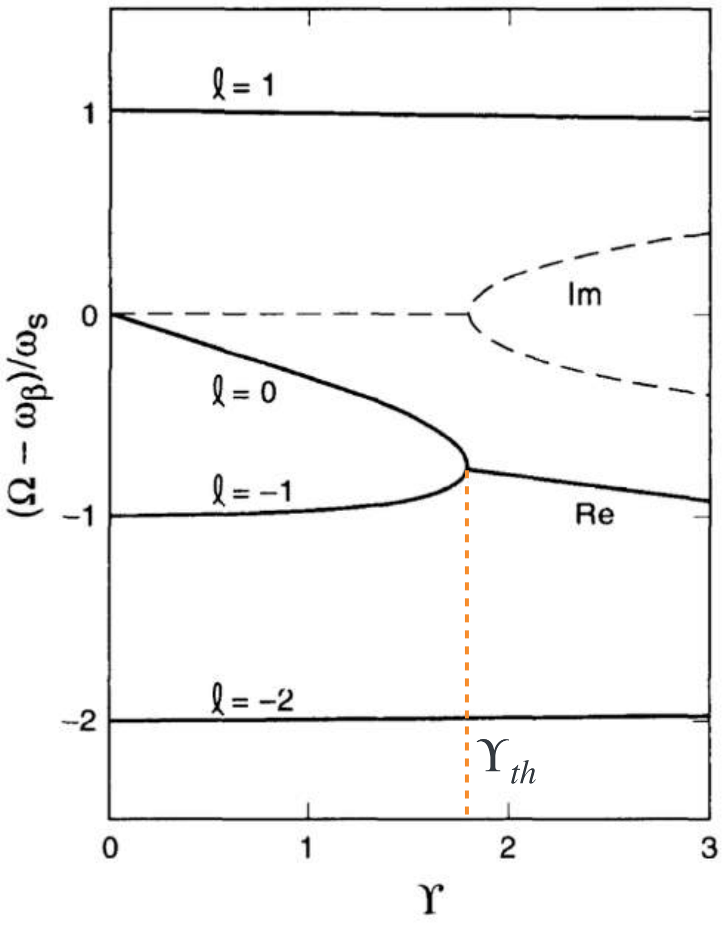
+ Injection/Extraction Kickers
+ Collimators
+ Beam Pipe Taperings
+ Any discontinuity…

A. W. Chao, "Physics of Collective Beam Instabilities in High Energy Accelerators", New York, NY, USA: Wiley, 1993

Motivation & Introduction: TMCI

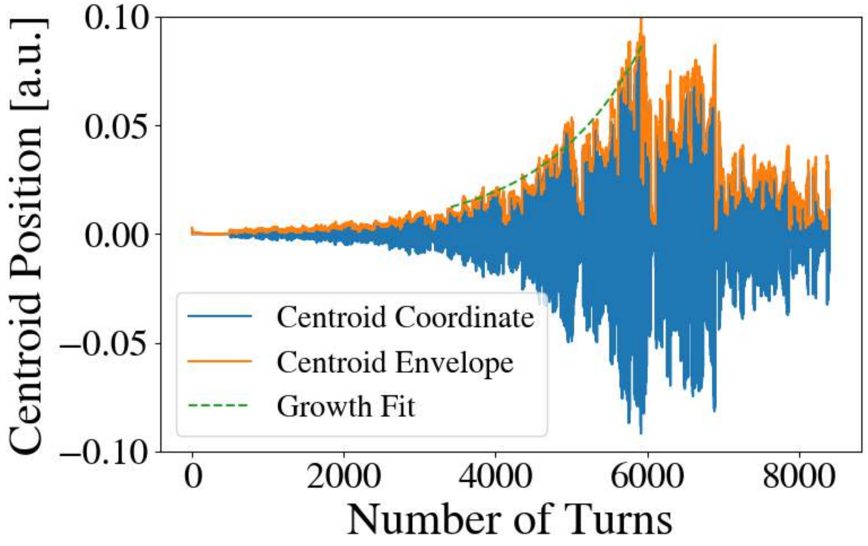
The beam has dipole modes of oscillation. When modes couple, a **Transverse Mode Coupling Instability** (**TMCI**) is born, with exponential growth characterized by growth rate τ^{-1}.





(Intensity Parameter)

$\mathrm{Im}\ \Omega > 0$

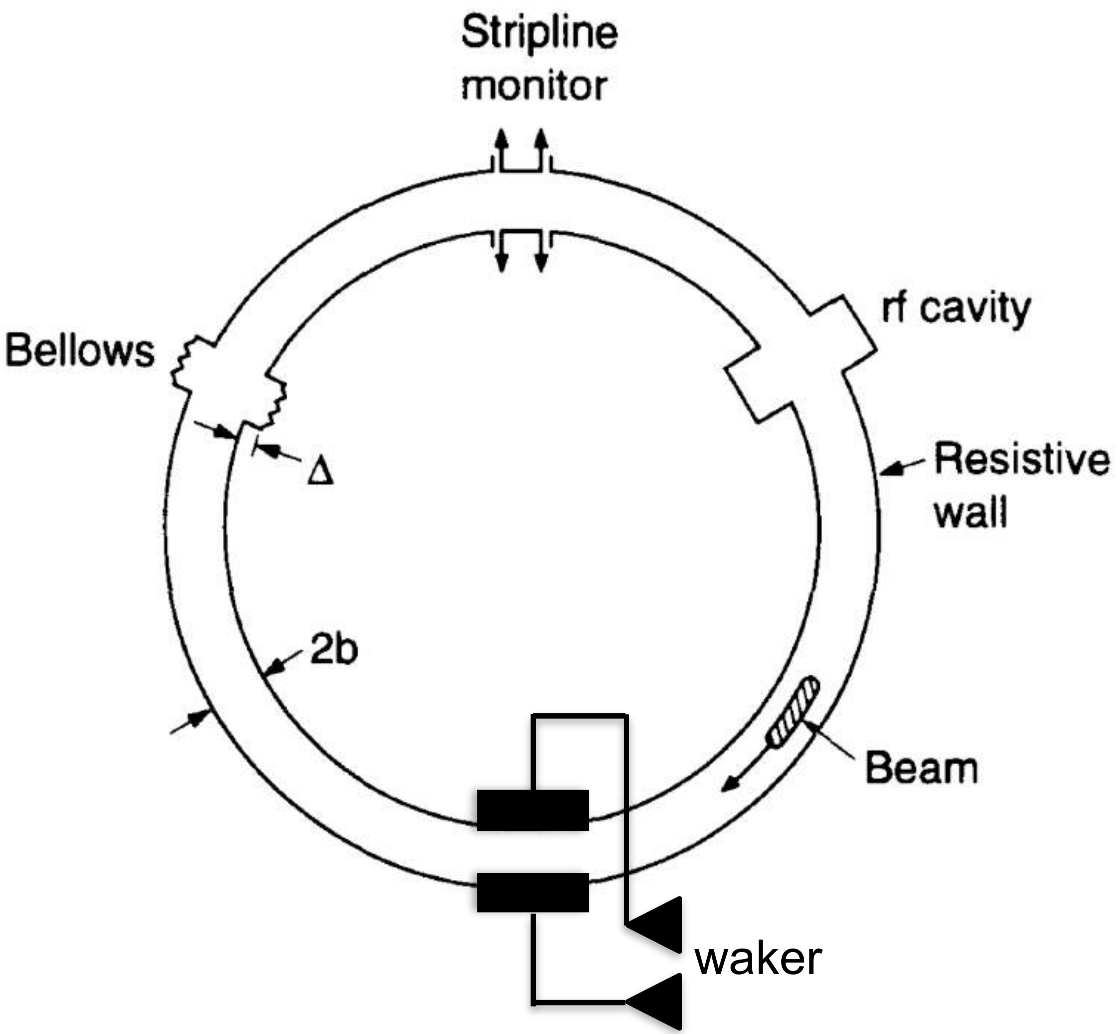


In order to model all of this we need to go and solve for our accelerator:

- The Vlasov Equation (No Space Charge)
- The Vlasov Equation + Poisson Equation (With Space Charge)

WAKER Experiment at Fermilab's Recycler Ring

The **WAKER experiment** at the Fermilab Recycler investigates the interplay between **space charge and transverse wakefields** in high-intensity proton beams.



In order to understand the experiments from the WAKER, we first need to understand the **naturally-occurring instabilities** in the Recycler with WAKER system turned off. That's where this work comes in.

Ainsworth, R., Burov, A., Eddy, N., & Semenov, A. (2021). A DEDICATED WAKE-BUILDING FEEDBACK SYSTEM TO STUDY SINGLE BUNCH INSTABILITIES IN THE PRESENCE OF STRONG SPACE CHARGE. Proceedings of HB'21, Batavia, IL, USA, October 2021, pp. 135–139.

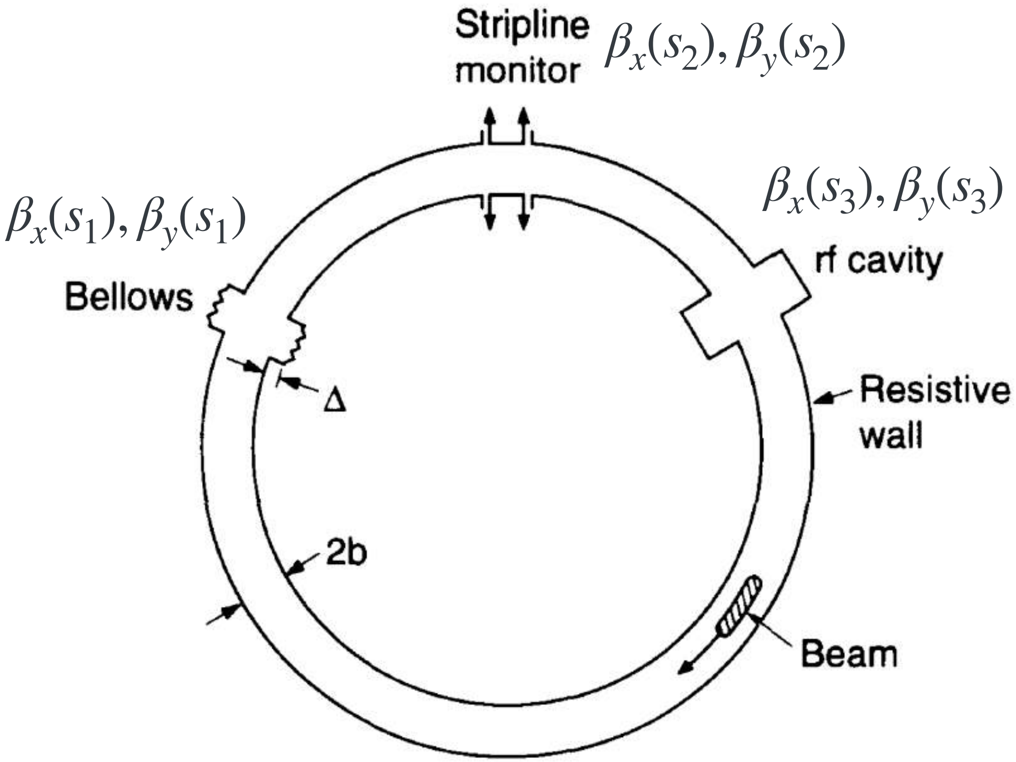
Mohsen, O., Ainsworth, R., & Eddy, N. (2022). WAKER EXPERIMENTS AT FERMILAB RECYCLER RING. Proceedings of NAPAC'22, Albuquerque, NM, USA, 2022, pp. 124–127.

02

Theory & Simulations (Part 1)

Single-Kick Model

The effective impedance model of the machine can be obtained by summing all the contributions from impedance sources around the machine into one **single-kick** and assume **smooth focusing**.



$$\langle\beta_x\rangle Z_x^{(eff)} = \sum_j \beta_x(s_j)\, Z_{x,j}$$

$$\langle\beta_y\rangle Z_y^{(eff)} = \sum_j \beta_y(s_j)\, Z_{y,j}$$

Distributed Impedance, e.g., Resistive Wall

$$\langle\beta_x\rangle Z_x^{(eff)} = Z_{x,j}\left(\frac{1}{C}\int_0^C ds\, \beta_x(s_j)\right)$$

Smooth focusing in the transverse directions:

$$Q_x = \frac{R_0}{\langle\beta_x\rangle}$$

$$Q_y = \frac{R_0}{\langle\beta_y\rangle}$$

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Single-Kick Model

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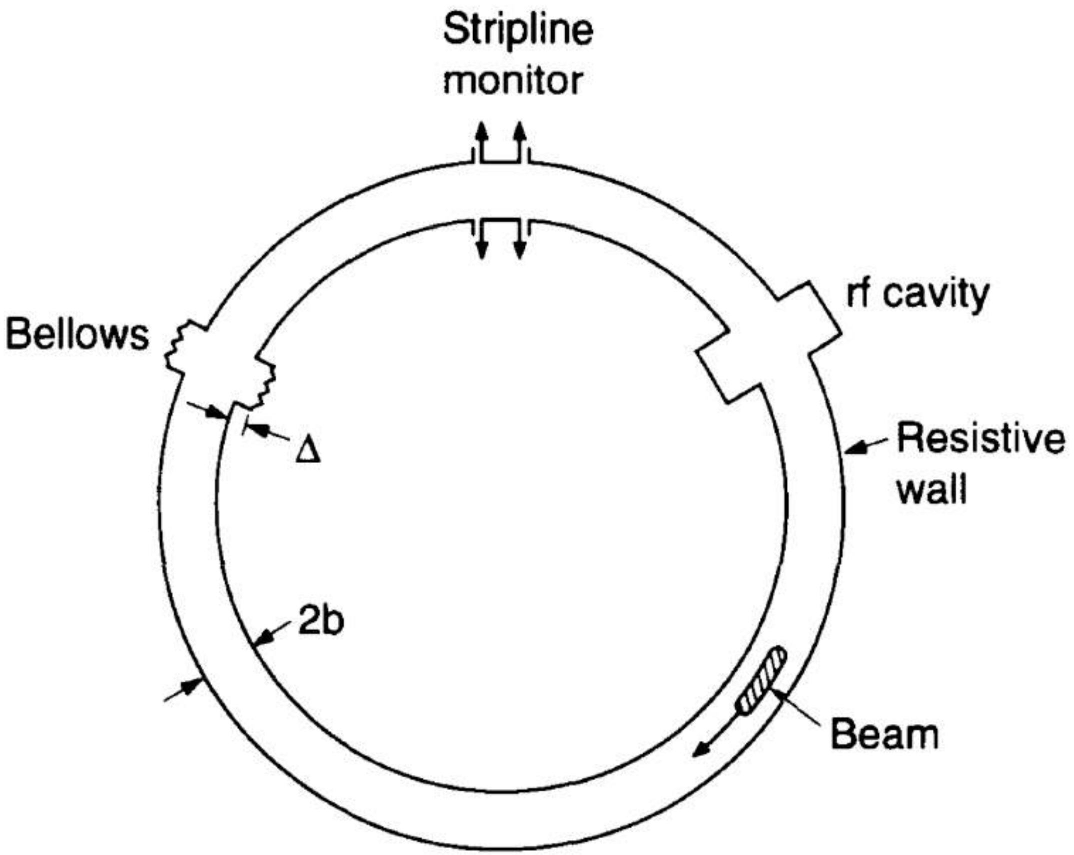
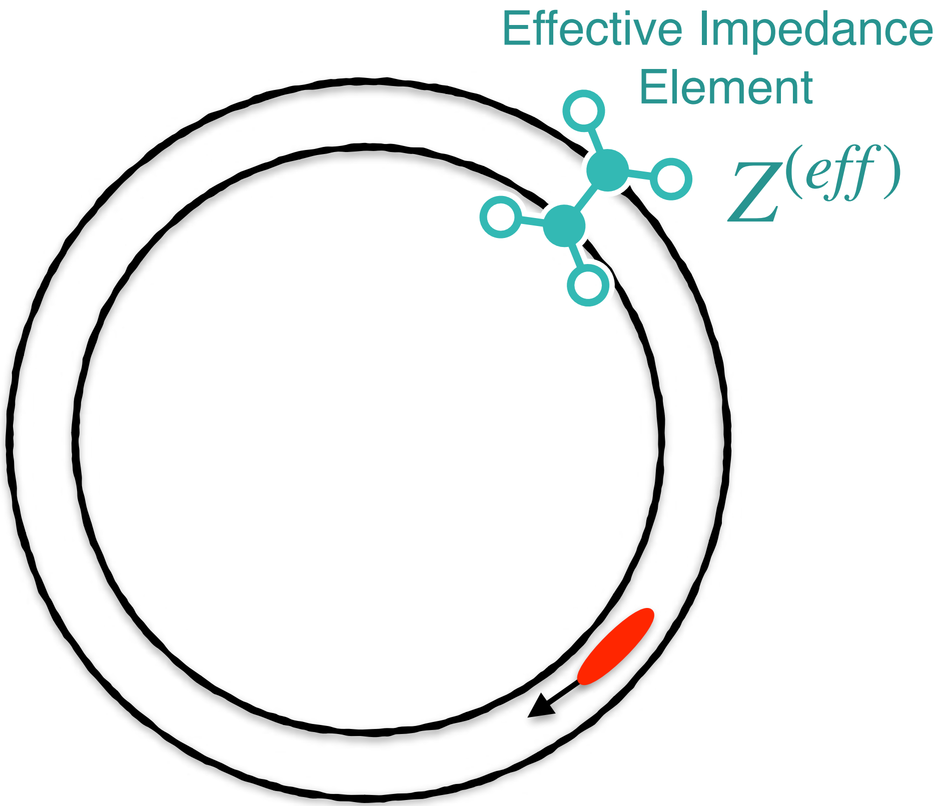


Complex accelerator model with
all types of impedance and lattice functions

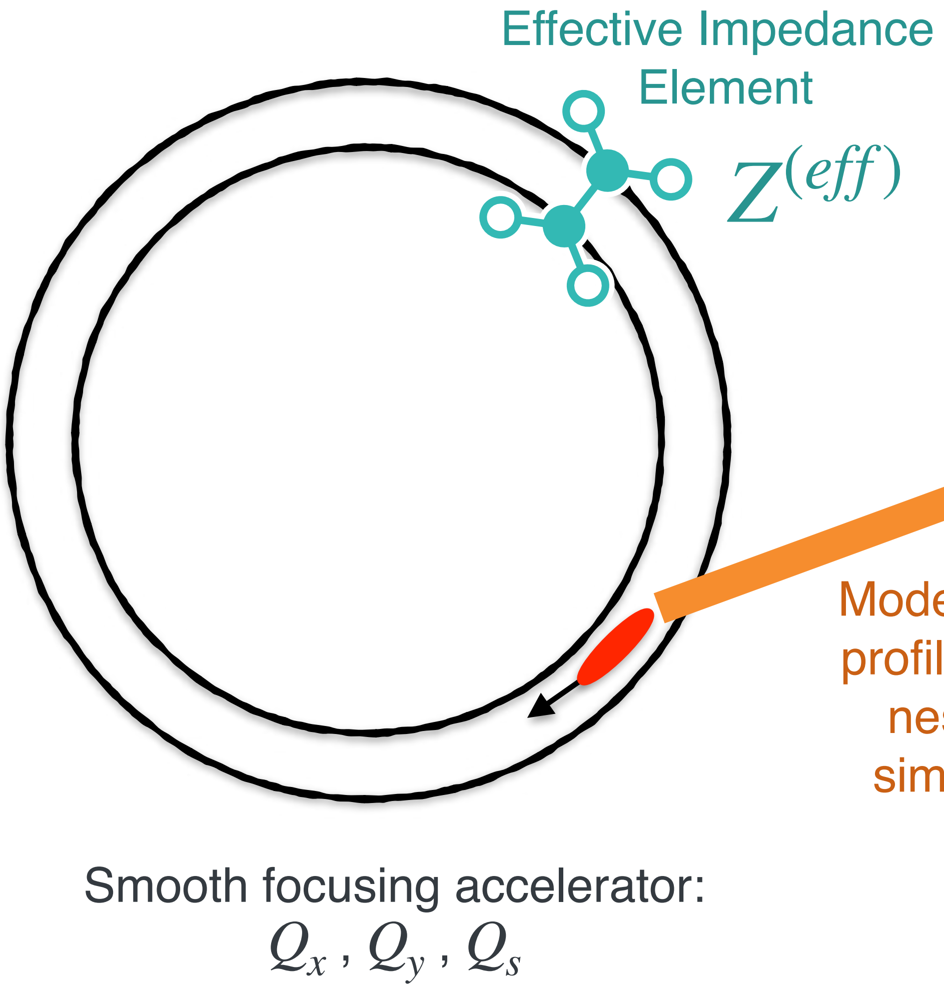


Smooth focusing accelerator:
Q_x , Q_y , Q_s

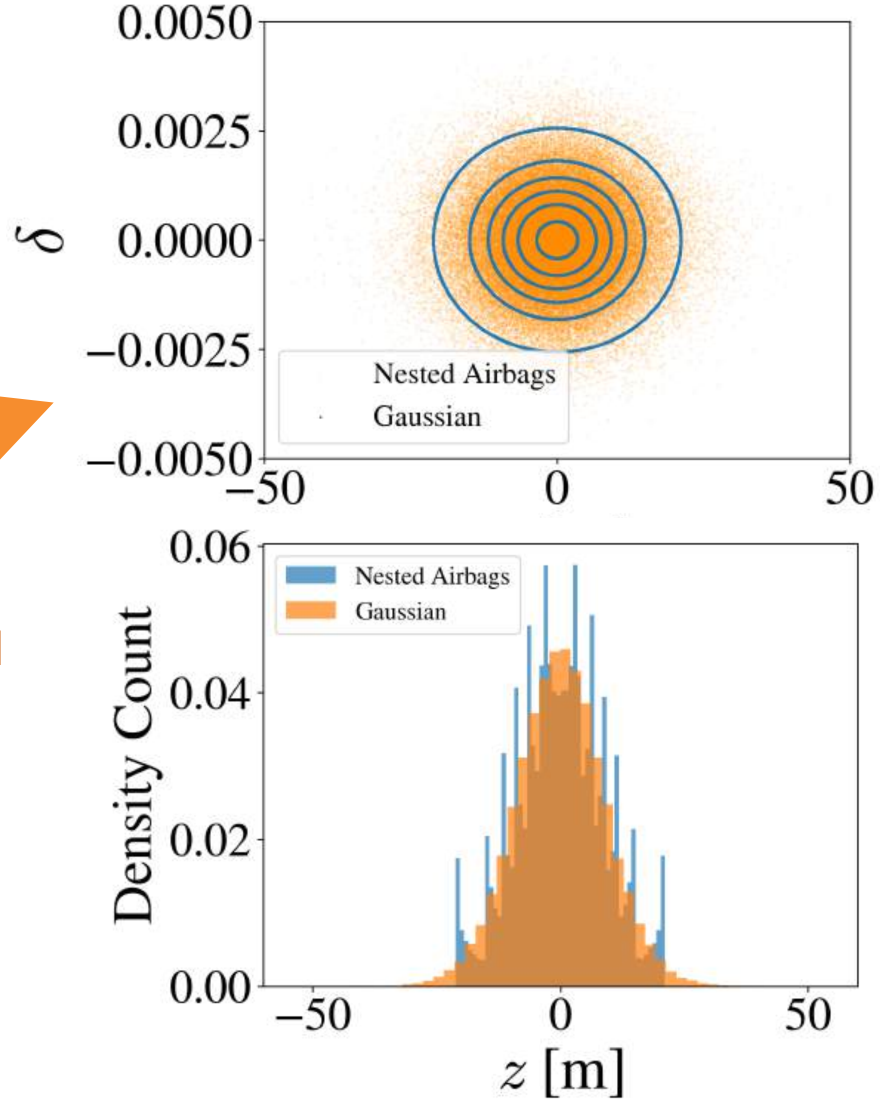
Single-Kick and NHT Model

A. Burov, "Nested head-tail Vlasov solver", PRAB 17, 021007 (2014)

Burov's NHT (**Nested Head-Tail**) Model can help us solve the Vlasov equation with semi-analytical methods for the single-bunch case and no space charge.



Model the longitudinal profile of the beam as nested airbags to simplify the Vlasov equation



Vlasov Equation Simplification

A. Burov, "Nested head-tail Vlasov solver", PRAB 17, 021007 (2014)

Burov's NHT (**Nested Head-Tail**) Model reduces the solution of the Vlasov equation to an eigenvalue problem for arbitrary impedances.

VLASOV EQUATION

$$\frac{\partial\psi}{\partial s} + y'\frac{\partial\psi}{\partial y} + p_y'\frac{\partial\psi}{\partial p_y} + z'\frac{\partial\psi}{\partial z} + \delta'\frac{\partial\psi}{\partial\delta} = 0$$

SINGLE PARTICLE EQUATIONS

$$y' = p_y$$

$$p_y' = -\left(\frac{2\pi f_0 Q_\beta}{c}\right)^2 y + \frac{1}{E}F_y(z,s)$$

$$z' = \eta\delta$$

$$\delta' = -\frac{1}{\eta}\left(\frac{2\pi f_0 Q_s}{c}\right)^2 z + \frac{y}{E}\frac{\partial F_y(z,s)}{\partial z}$$

SINGLE-KICK NHT EIGENSYSTEM

$$\left(\frac{\Omega - Q_\beta}{Q_s}\right)\mathbf{X} = \hat{\mathbf{S}}\cdot\mathbf{X} - i\hat{\mathbf{Z}}\cdot\mathbf{X}$$

$$\hat{S}_{lm\alpha\beta} = l\delta_{lm}\delta_{\alpha\beta}$$

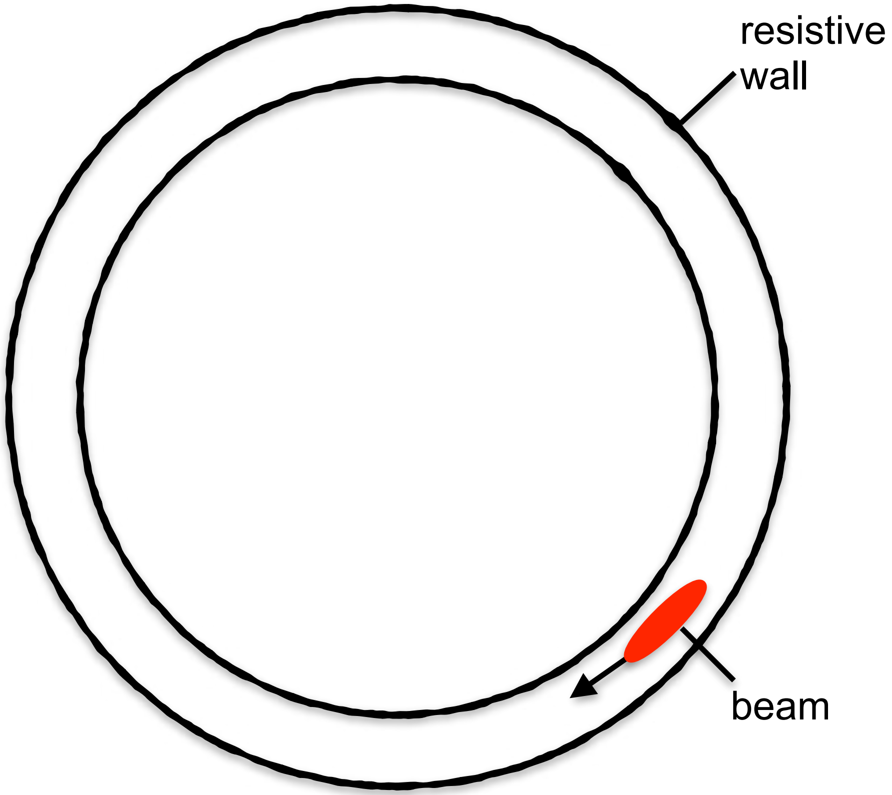
$$\hat{Z}_{lm\alpha\beta} = i^{l-m}\frac{\kappa}{n_r}\int_{-\infty}^{\infty} d\omega\, Z_\perp(\omega)J_l\left(\omega\tau_\alpha - \chi_\alpha\right)J_m\left(\omega\tau_\beta - \chi_\beta\right)$$

$Z_\perp(\omega)$: Transverse impedance
l, m : Azimuthal head-tail harmonics indices
α, β : Radial airbag indices
τ_α : Radius of the α airbag
J_l : Bessel Function of order l
n_r : Number of airbags
χ_α : Head-tail phase of airbag α
κ : Wake parameter

Elliptical Resistive Wall Beam Pipe

R. L. Gluckstern, J. van Zeijts, and B. Zotter, Coupling impedance of beam pipes of general cross section, Phys. Rev. E 47, 656 (1993).

The Recycler Ring has an elliptical beam pipe. The resistive wall wake will be mediated by the geometry of the beam pipe.



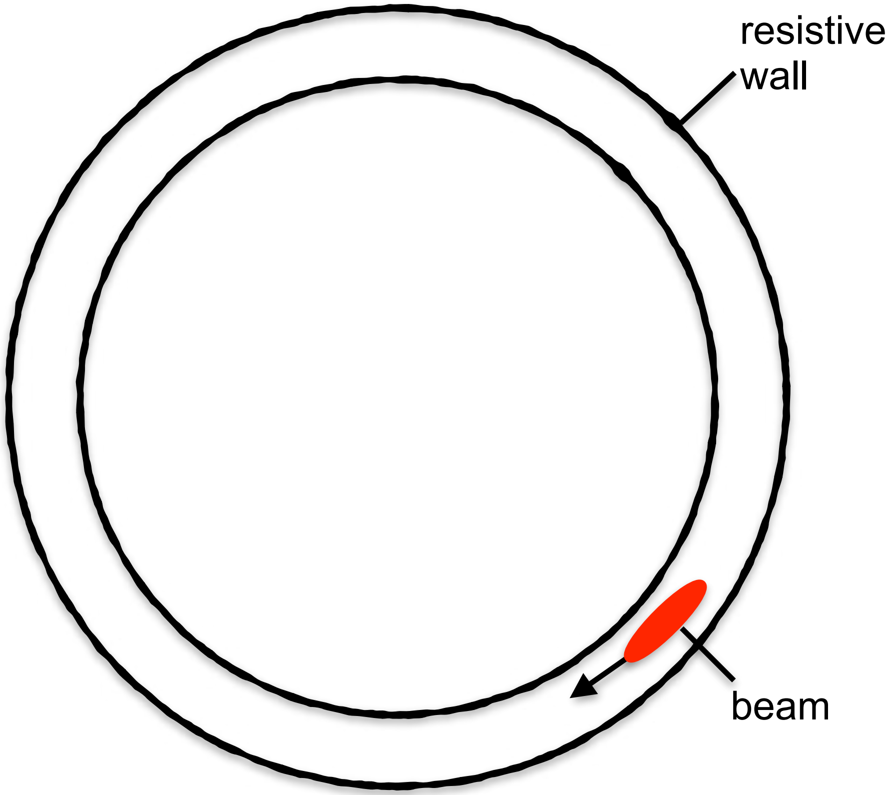
$$Z_{\perp}^{RW}(\omega) = R_W \times \begin{cases} \frac{1-i}{\omega^{1/2}} & \text{for } \omega > 0 \\ \frac{1+i}{-|\omega|^{1/2}} & \text{for } \omega < 0 \end{cases}$$

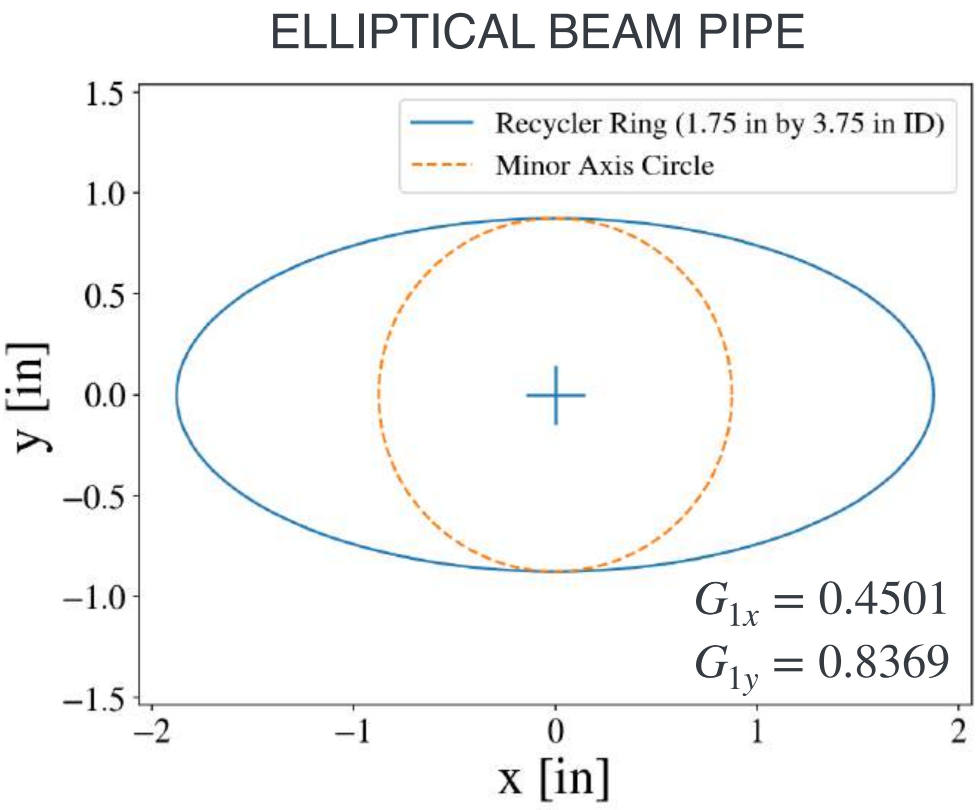
ELLIPTICAL BEAM PIPE

Elliptical Resistive Wall Beam Pipe

R. L. Gluckstern, J. van Zeijts, and B. Zotter, Coupling impedance of beam pipes of general cross section, Phys. Rev. E 47, 656 (1993).

The Recycler Ring has an elliptical beam pipe. The resistive wall wake will be mediated by the geometry of the beam pipe. The **Yokoya factors** quantify the impedance difference to circular pipe.





$$Z_{\perp}^{RW}(\omega) = R_W \times \begin{cases} \frac{1-i}{\omega^{1/2}} & \text{for } \omega > 0 \\ \frac{1+i}{-|\omega|^{1/2}} & \text{for } \omega < 0 \end{cases}$$

$$Z_x^{RW} = G_{1x} Z_{\perp}^{RW}$$

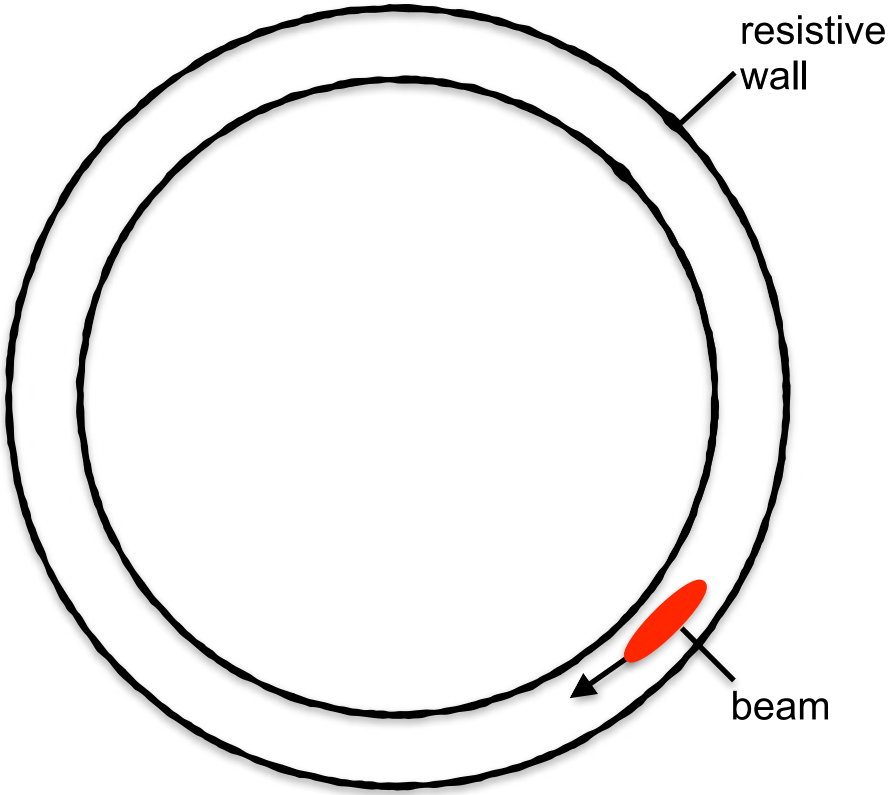
$$Z_y^{RW} = G_{1y} Z_{\perp}^{RW}$$

Yokoya Factors: Difference to Circular Beam Pipe

Elliptical Resistive Wall Beam Pipe

R. L. Gluckstern, J. van Zeijts, and B. Zotter, Coupling impedance of beam pipes of general cross section, Phys. Rev. E 47, 656 (1993).

With the impedance expression set, we can calculate the **impedance operator** that goes into calculating the eigenmodes in the NHT model. Analytical expressions allow fast calculations.



$$\hat{Z}_{lm\alpha\beta} = i^{l-m}\frac{\kappa}{n_r}\int_{-\infty}^{\infty} d\omega\, Z_{\perp}(\omega) J_l\left(\omega\tau_\alpha - \chi_\alpha\right) J_m\left(\omega\tau_\beta - \chi_\beta\right)$$

$$\boxed{\hat{Z}^{RW}_{lm\alpha\beta} = i^{l-m}\frac{\kappa}{n_r} R_W \cdot \Phi^{(1/2)}_{lm\alpha\beta}\left[(1-i) - (1+i)(-1)^{l+m}\right]}$$

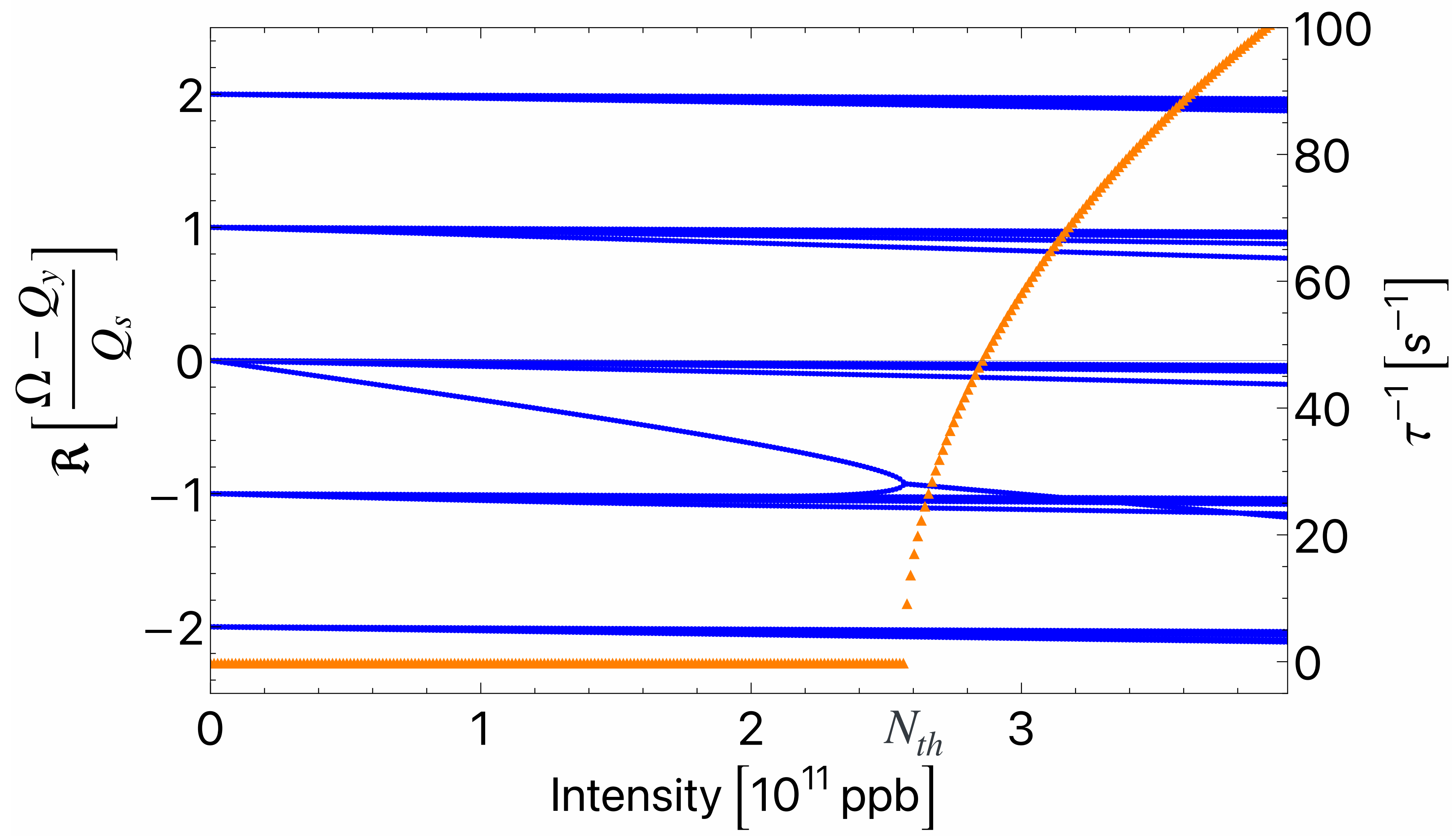
$$\Phi^{(1/2)}_{lm\alpha\beta} = \begin{cases} 0 & \text{if } l = m = 0 \\ \frac{2}{\sqrt{\pi}}\frac{\Gamma\left(\frac{5}{4}\right)\Gamma\left(l+\frac{1}{4}\right)}{\sqrt{\tau_\alpha}\,\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{3}{4}+l\right)} & \text{if } l = m \neq 0 \text{ and } \tau_\beta = \tau_\alpha \\ \frac{1}{\sqrt{2}}\frac{\tau_\alpha^l}{\tau_\beta^{l+1/2}}\frac{\Gamma\left(\frac{1}{4}+\frac{l+m}{2}\right)}{\Gamma\left(\frac{3}{4}+\frac{l-m}{2}\right)}\,{}_2F_1\left(\frac{1}{4}+\frac{l-m}{2}, \frac{1}{4}+\frac{l+m}{2}, 1+l, \left(\frac{\tau_\alpha}{\tau_\beta}\right)^2\right) & \text{if } \tau_\beta > \tau_\alpha \\ \frac{1}{\sqrt{2}}\frac{\tau_\beta^m}{\tau_\alpha^{m+1/2}}\frac{\Gamma\left(\frac{1}{4}+\frac{m+l}{2}\right)}{\Gamma\left(\frac{3}{4}+\frac{m-l}{2}\right)}\,{}_2F_1\left(\frac{1}{4}+\frac{m-l}{2}, \frac{1}{4}+\frac{m+l}{2}, 1+m, \left(\frac{\tau_\beta}{\tau_\alpha}\right)^2\right) & \text{if } \tau_\beta < \tau_\alpha \\ \sqrt{\frac{\pi}{2\tau_\alpha}}\frac{\Gamma\left(\frac{1}{4}+\frac{l+m}{2}\right)}{\Gamma\left(\frac{3}{4}+\frac{l-m}{2}\right)\Gamma\left(\frac{3}{4}+\frac{m-l}{2}\right)\Gamma\left(\frac{3}{4}+\frac{l+m}{2}\right)} & \text{if } l \neq m \text{ and } \tau_\beta = \tau_\alpha. \end{cases}$$

$$Z_{\perp}^{RW}(\omega) = R_W \times \begin{cases} \frac{1-i}{\omega^{1/2}} & \text{for } \omega > 0 \\ \frac{1+i}{-|\omega|^{1/2}} & \text{for } \omega < 0 \end{cases}$$

Solution to NHT Model with Resistive Wall Wake

$$\left(\frac{\Omega - Q_\beta}{Q_s}\right)\mathbf{X} = \hat{\mathbf{S}} \cdot \mathbf{X} - i\hat{\mathbf{Z}}_{RW} \cdot \mathbf{X}$$

Solution with 10 airbags and 41 azimuthal modes yields a threshold of around 2.6×10^{11} protons per bunch. After the modes couple an instability with growth rate τ^{-1} appears. **Zero chromaticity**.

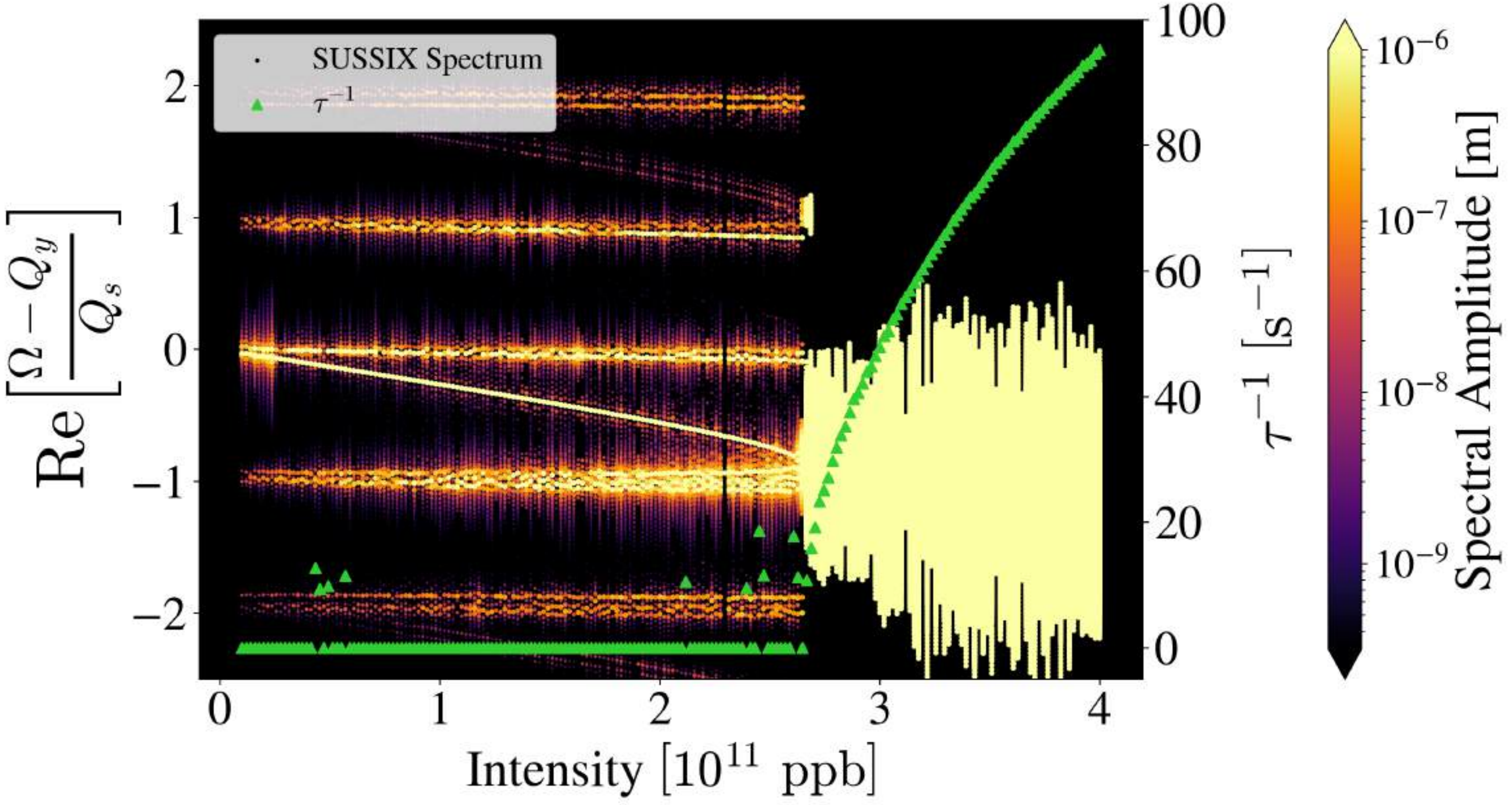


PyHEADTAIL Simulations

A. Oeftiger, An Overview of PyHEADTAIL, Tech. Rep. (CERN, 2019).

SUSSIX: A computer code for frequency analysis of non linear betatron motion (2023). CERN.

PyHEADTAIL is a macroparticle tracking code designed specifically to **simulate collective effects in circular accelerators.** Combined with **SUSSIX** to get main oscillation frequencies.

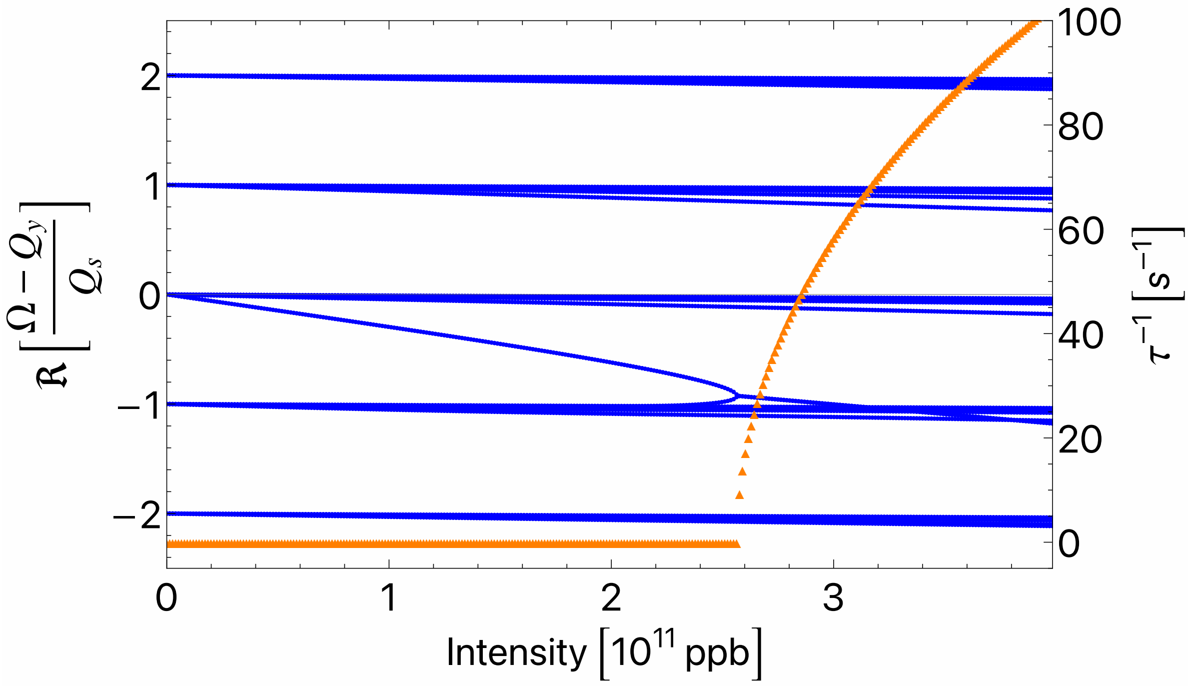


Parameter	Value	Unit
Circumference C	3319.4	m
Machine Mean Radius R_0	528.30	m
Momentum	8.835	GeV/c
Relativistic Factor γ	9.47	
Revolution Period T_0	11.1	μs
Revolution Frequency f_0	89.8	kHz
RF Frequency	2.5	MHz
RF Voltage	80	kV
Synchrotron Tune Q_s	0.000595	
Slip Factor η	-8.6×10^{-3}	
Superperiodicity	2	
Horizontal Tune Q_x	25.4601	
Vertical Tune Q_y	24.412	
Horizontal Chromaticity ξ_x	0	
Vertical Chromaticity ξ_y	0	
95% Normalized Emittance	15	π mm mrad
Bunch Length σ_z	9	m
95% Longitudinal Emittance	3.64	eV s

Theory & Simulations (NHT vs. PyHEADTAIL)

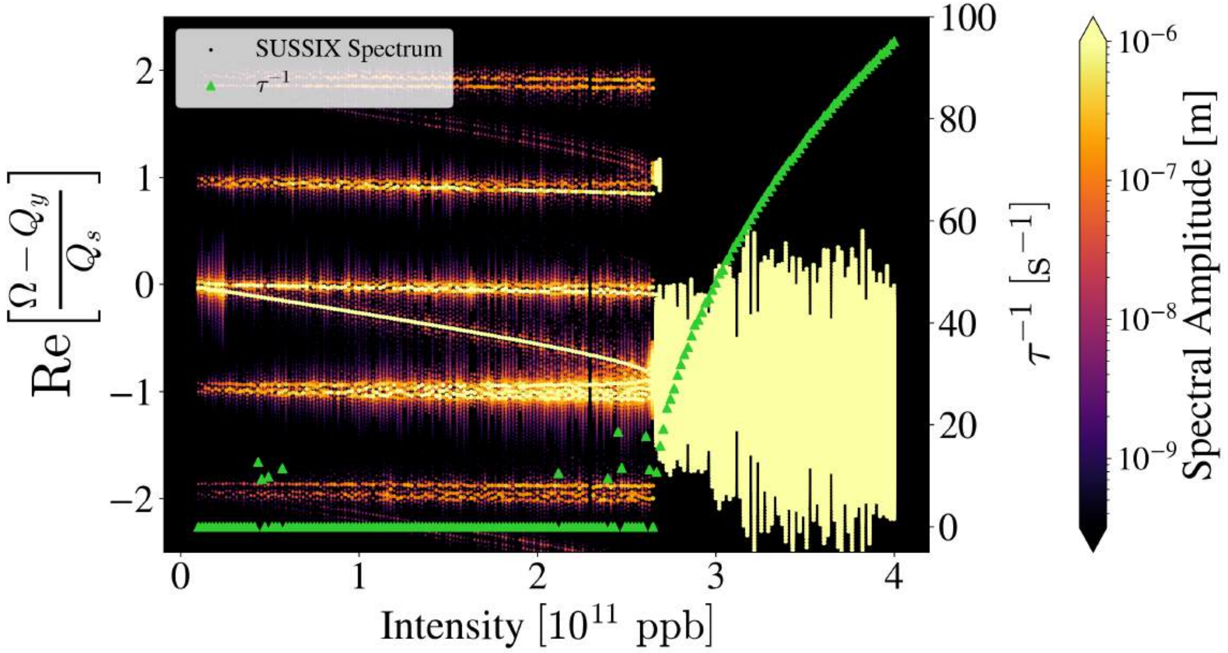
With theory and simulations, only including a resistive wall interaction, we can have an idea of what to expect in experiments. It's also a good way to benchmark analytical results.

NHT Model



~10 seconds on personal laptop

PyHEADTAIL



~3 days on EAF cluster

03

Experiments

Experiment Nominal Parameters

Knobbed several parameters to try and observe **instability thresholds.**

Parameter	Value	Unit
Horizontal Tune Q_x	25.4601	
Vertical Tune Q_y	24.412	
Horizontal Chromaticity ξ_x	0	
Vertical Chromaticity ξ_y	0	
95% Normalized Emittance	15	π mm mrad
Bunch Length σ_z	9	m
95% Longitudinal Emittance	3.64	eV s

- +

Intensity [protons per bunch]

Parameter	Value	Unit
Circumference C	3319.4	m
Machine Mean Radius R_0	528.30	m
Momentum	8.835	GeV/c
Relativistic Factor γ	9.47	
Revolution Period T_0	11.1	μs
Revolution Frequency f_0	89.8	kHz
RF Frequency	2.5	MHz
RF Voltage	80	kV
Synchrotron Tune Q_s	0.000595	
Slip Factor η	-8.6 $\times 10^{-3}$	

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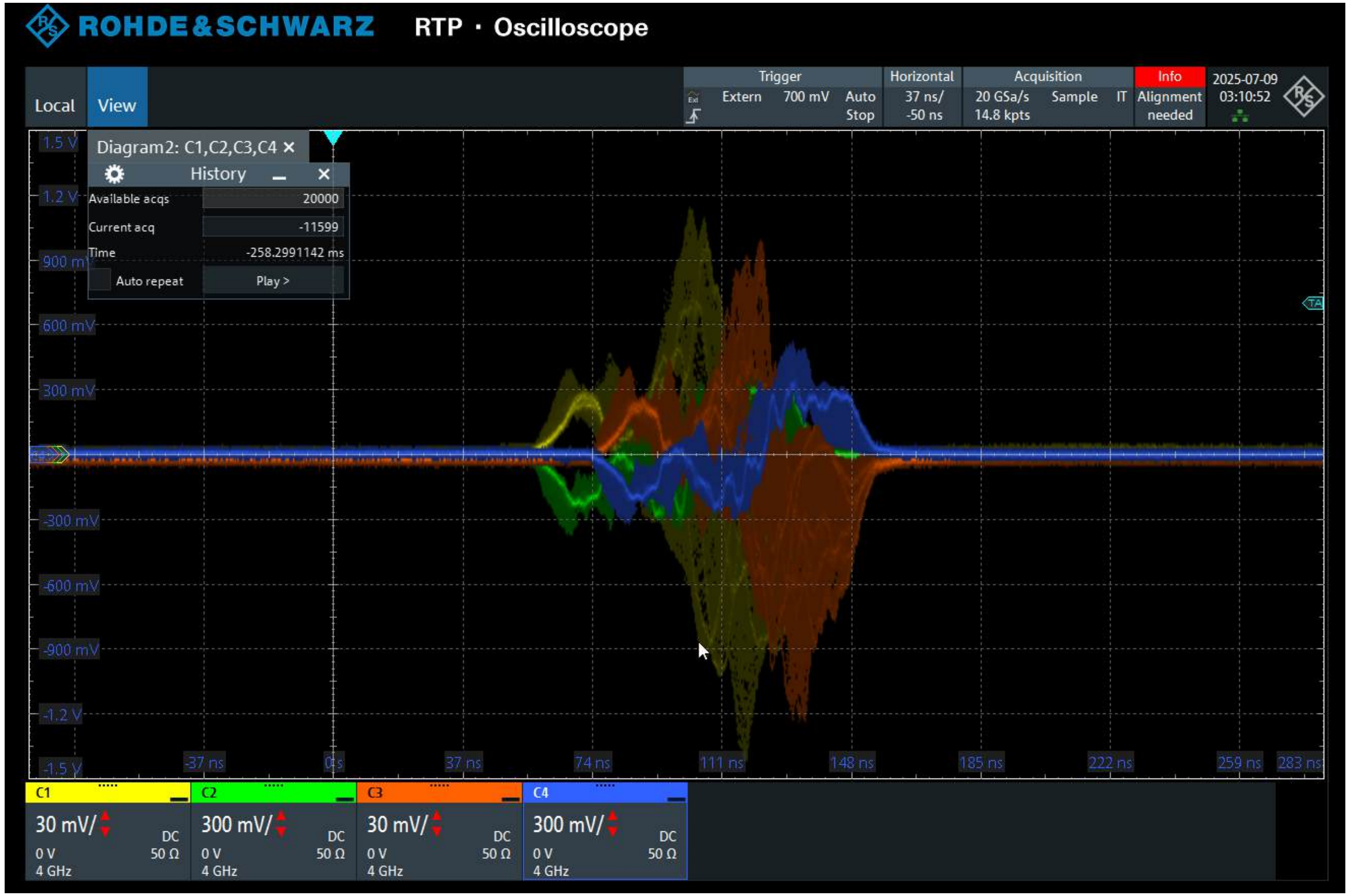
Experimental Setup

Used **stripline pick-up** to reconstruct **dipole oscillations** and longitudinal beam profile. Saved data to oscilloscope and accessed it for offline analysis.

- Long stripline pickup to capture dipole motion in both planes
  - A+B (Sum: Longitudinal Profile)
  - A-B (Difference: Centroid Position)
- Transverse dampers and WAKER system are turned off.
- Got data every other turn with our trigger system.
- 2.5 MHz RF station is used (as opposed to 53 MHz used in high-intensity neutrino operation).
  - **Relatively long bunches allowing to capture intrabunch motion**

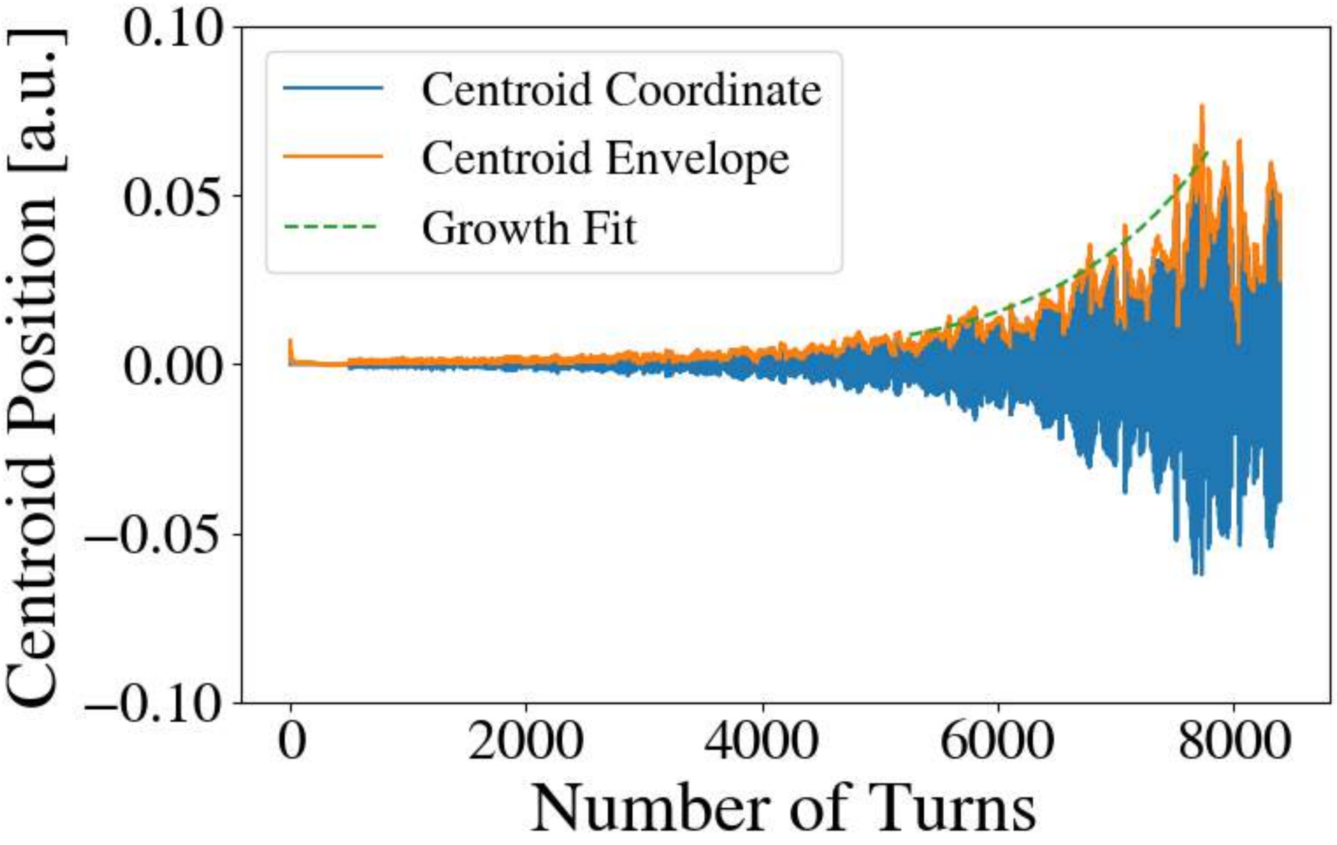


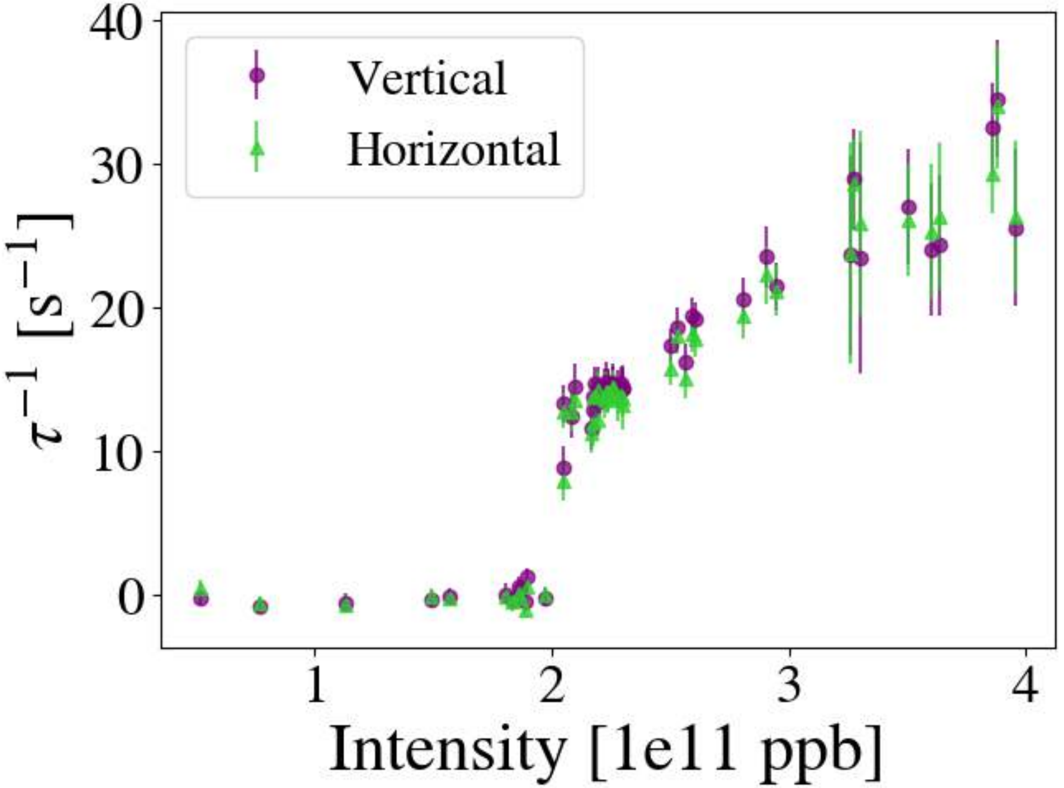
Vertical A-B	Vertical A+B	Horizontal A-B	Horizontal A+B

Experimental Setup: Instability Example

Experimental Observation #1: Growth Rate vs. Intensity

The instability has a sharp threshold with intensity. After the threshold, an instability appears in both planes with growth rates proportional to the intensity.

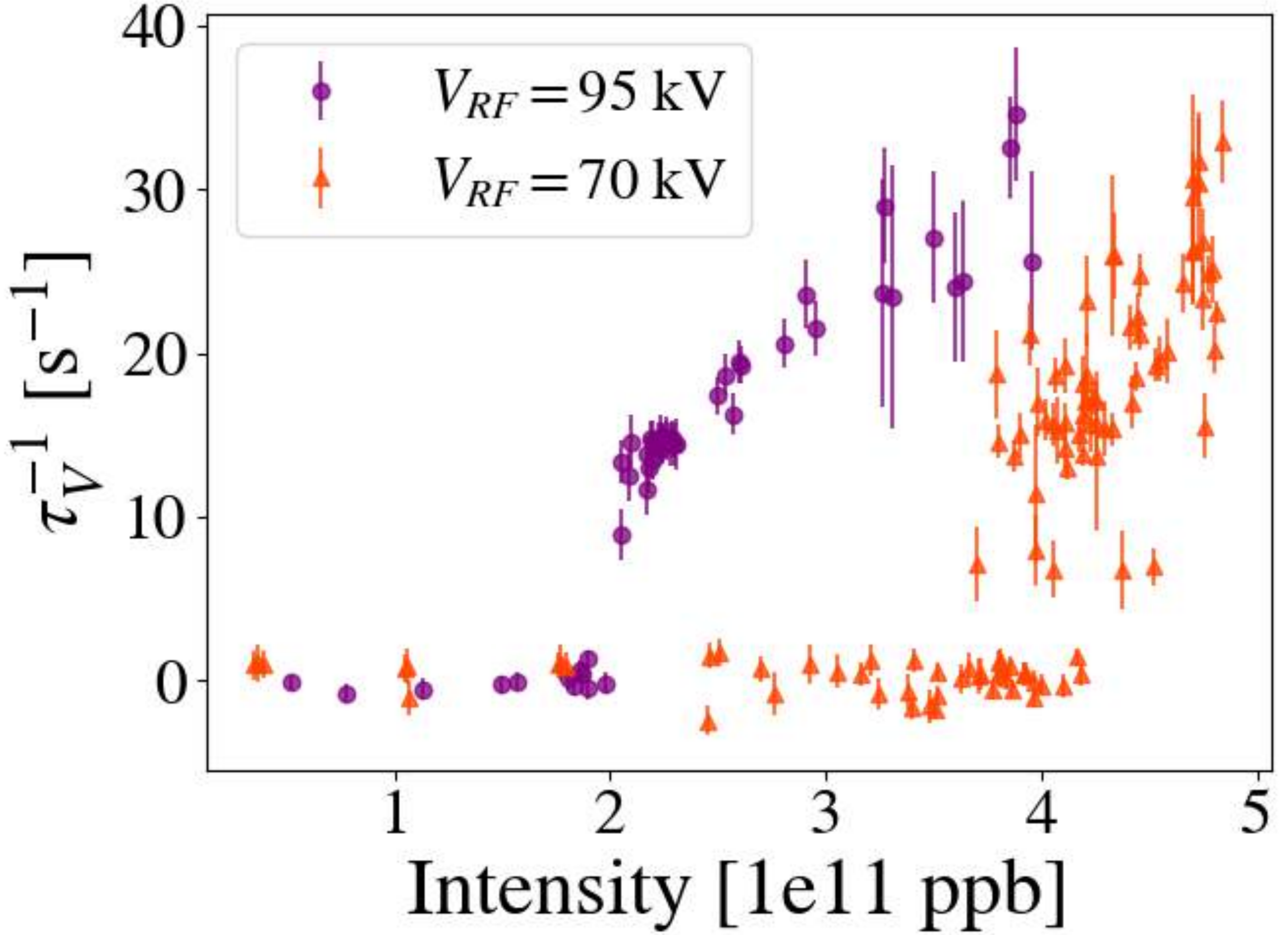




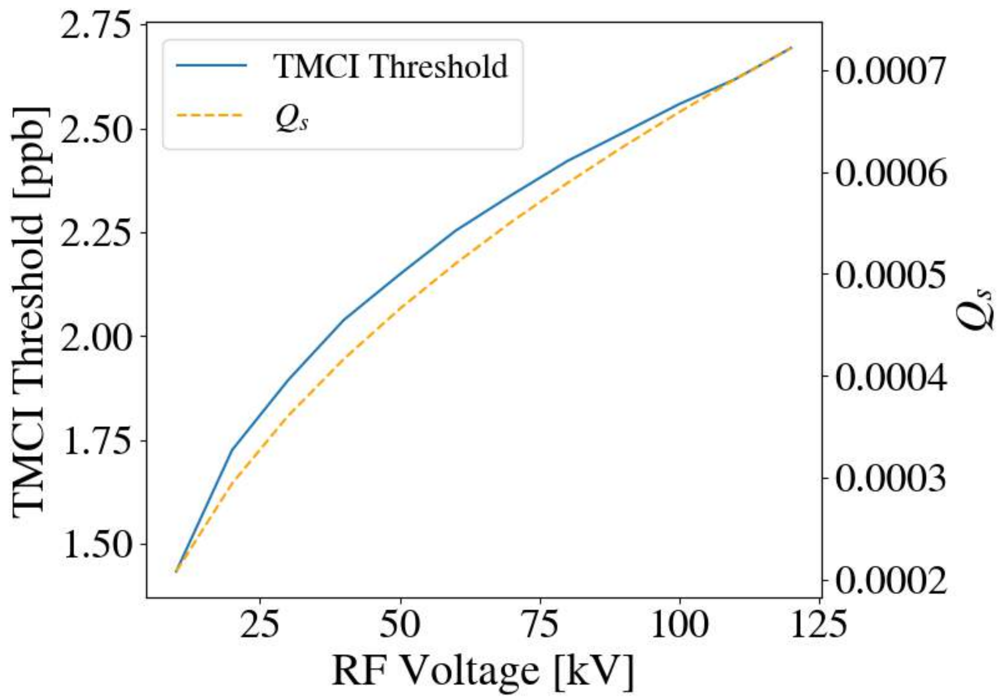
Experimental Observation #2: RF Voltage and Threshold

The instability threshold decreases when RF voltage is increased. This behavior is not predicted by our simple initial model with no space charge.

EXPERIMENTS

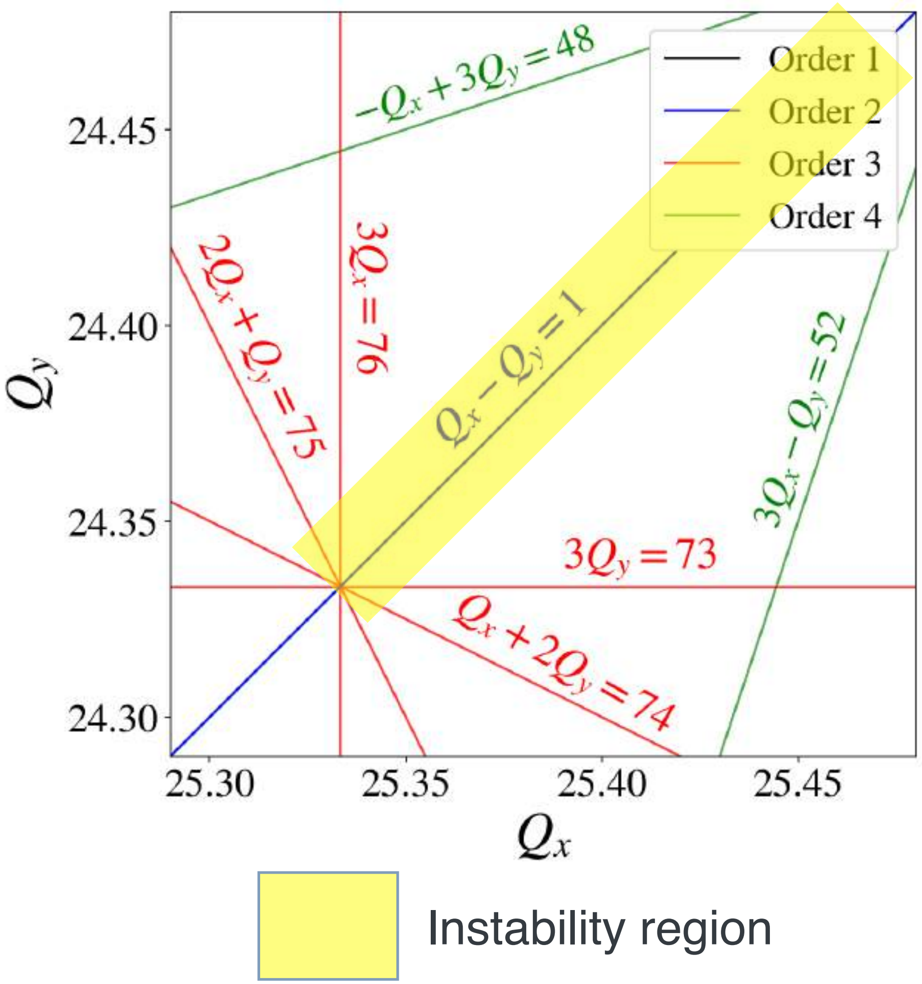


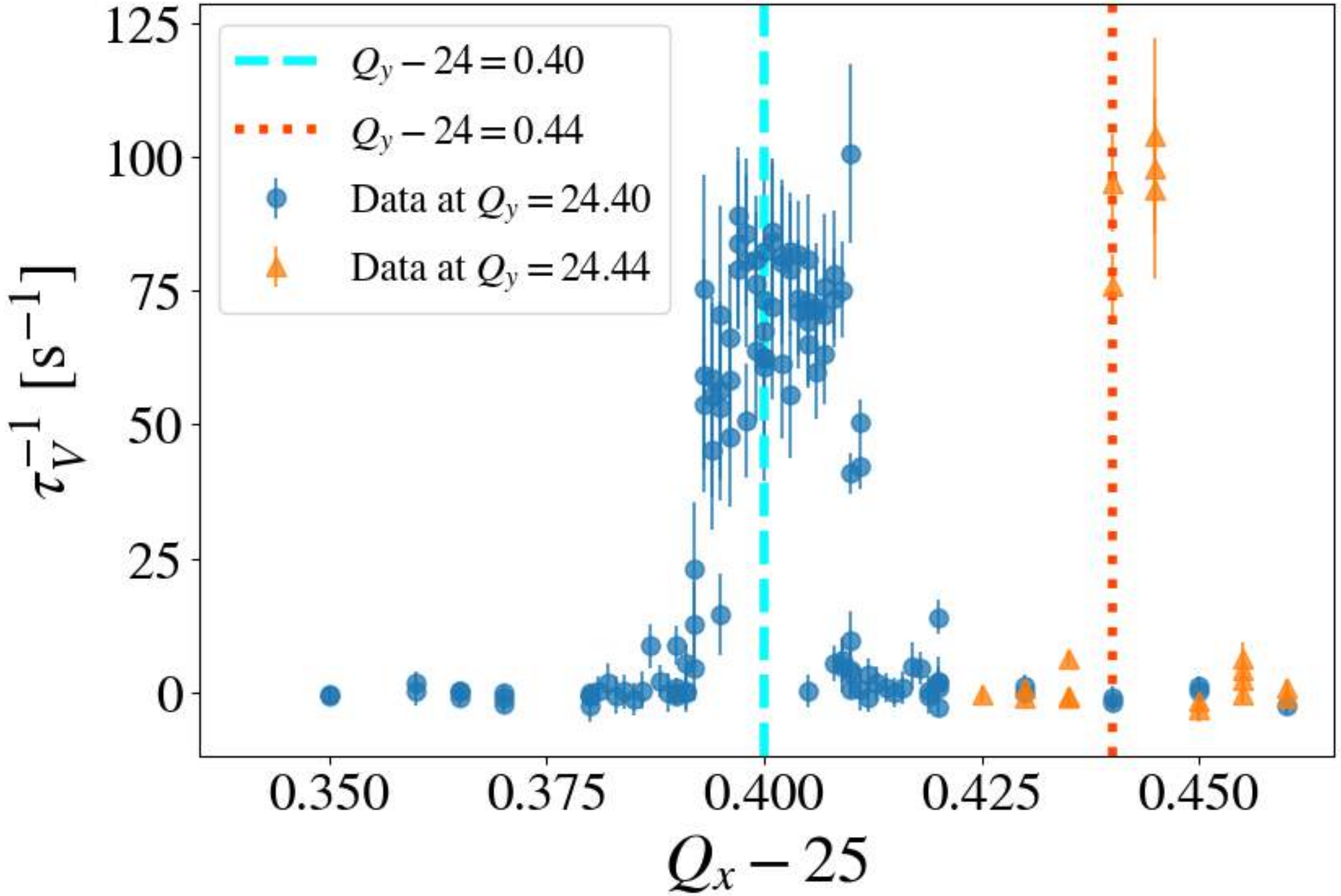
NHT Theory



Experimental Observation #3: Instability and Linear Coupling

The instability only appears close to the coupling line $Q_x - Q_y = 1$. The maximum intensity output from Booster does not allow us to measure the TMCI threshold outside of the coupling line.





We only measure an instability that resembles TMCI when we couple the tunes in the Recycler.

This is not predicted by our simple uncoupled resistive-wall model.

How to incorporate linear coupling into the model?

How to incorporate space charge into the model?

04

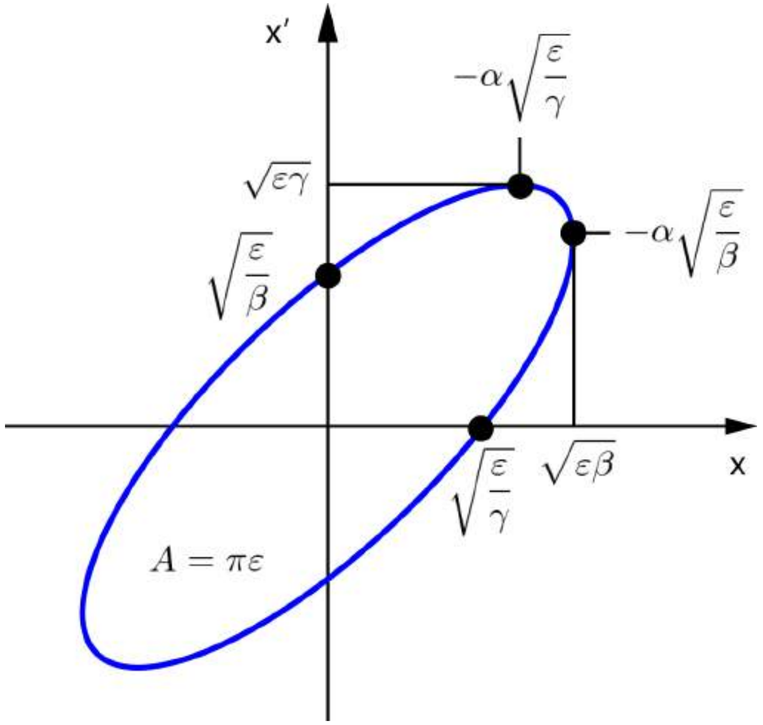
Theory & Simulations (Part 2: Linear Coupling)

Linear Coupling Formalism: Lebedev & Bogacz

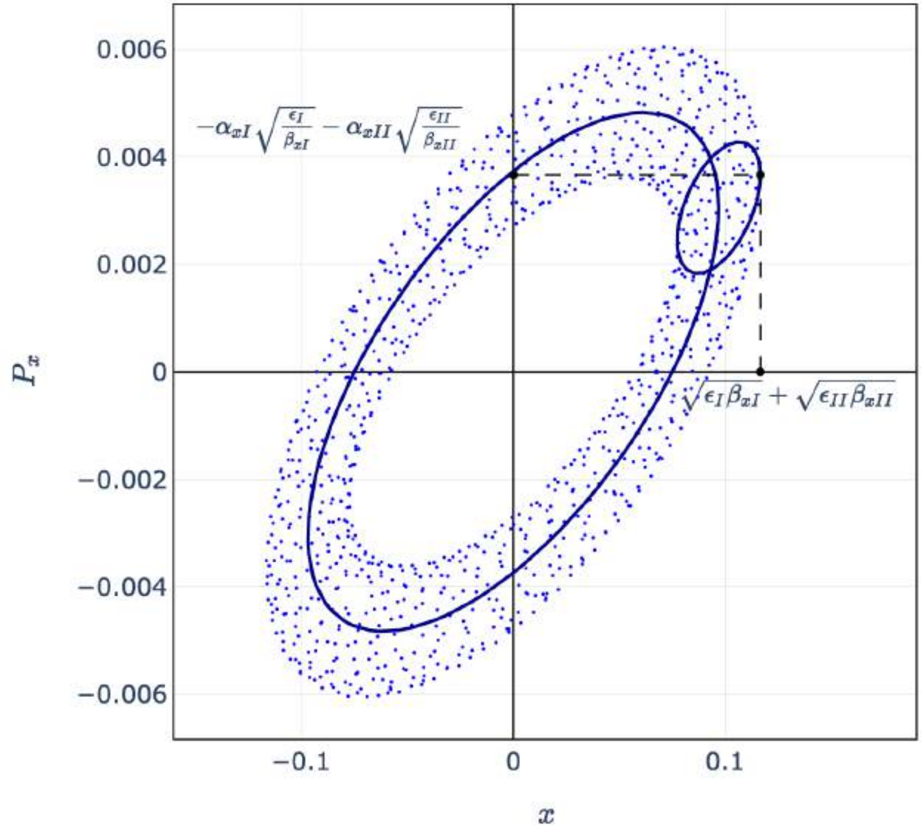
Coupling horizontal and vertical planes

V. A. Lebedev and S. A. Bogacz, Betatron motion with coupling of horizontal and vertical degrees of freedom, JINST 5 (10), P10010

Twiss Parameters (Uncoupled)



Lebedev & Bogacz Parameters (Coupled)



M. Vanwelde, et. al., Parametrizations of coupled betatron motion for strongly coupled lattices, JINST 18 (10), P10012

Linear Coupling Formalism: Lebedev & Bogacz

Coupling horizontal and vertical planes

M. Vanwelde, et. al., Parametrizations of coupled betatron motion for strongly coupled lattices, JINST 18 (10), P10012

V. A. Lebedev and S. A. Bogacz, Betatron motion with coupling of horizontal and vertical degrees of freedom, JINST 5 (10), P10010

Twiss Parameters (Uncoupled)

Uncoupled Lattice Functions
(6 free parameters)

$$\beta_x, \alpha_x, \mu_x$$

$$\beta_y, \alpha_y, \mu_y$$

Lebedev & Bogacz Parameters (Coupled)

Willeke & Ripken	Lebedev & Bogacz	Wolski
Principal lattice functions		
β_{xI}	β_{1x}	β_x
β_{yII}	β_{2y}	β_y
$\alpha_{xI} + \frac{R_1}{2}\sqrt{\beta_{xI}\beta_{yI}}\cos(\nu_1)$	α_{1x}	α_x
$\alpha_{yII} - \frac{R_2}{2}\sqrt{\beta_{xII}\beta_{yII}}\cos(\nu_2)$	α_{2y}	α_y
ϕ_{xI}	μ_1	μ_I
ϕ_{yII}	μ_2	μ_{II}
Non-principal lattice functions		
β_{xII}	β_{2x}	$\lvert\zeta_y\rvert^2$
β_{yI}	β_{1y}	$\lvert\zeta_x\rvert^2$
$\alpha_{xII} + \frac{R_1}{2}\sqrt{\beta_{xII}\beta_{yII}}\cos(\nu_2)$	α_{2x}	$-Re(\zeta_y\tilde{\zeta}_x)$
$\alpha_{yI} - \frac{R_2}{2}\sqrt{\beta_{xI}\beta_{yI}}\cos(\nu_1)$	α_{1y}	$-Re(\zeta_x\tilde{\zeta}_y)$
ϕ_{xII}	$\mu_1 - \nu_1$	$\mu_I + ph(\zeta_x)$
ϕ_{yI}	$\mu_2 - \nu_2$	$\mu_{II} + ph(\zeta_y)$

Coupled Lattice Functions
(10 free parameters)

Linear Coupling Formalism: Edwards & Teng

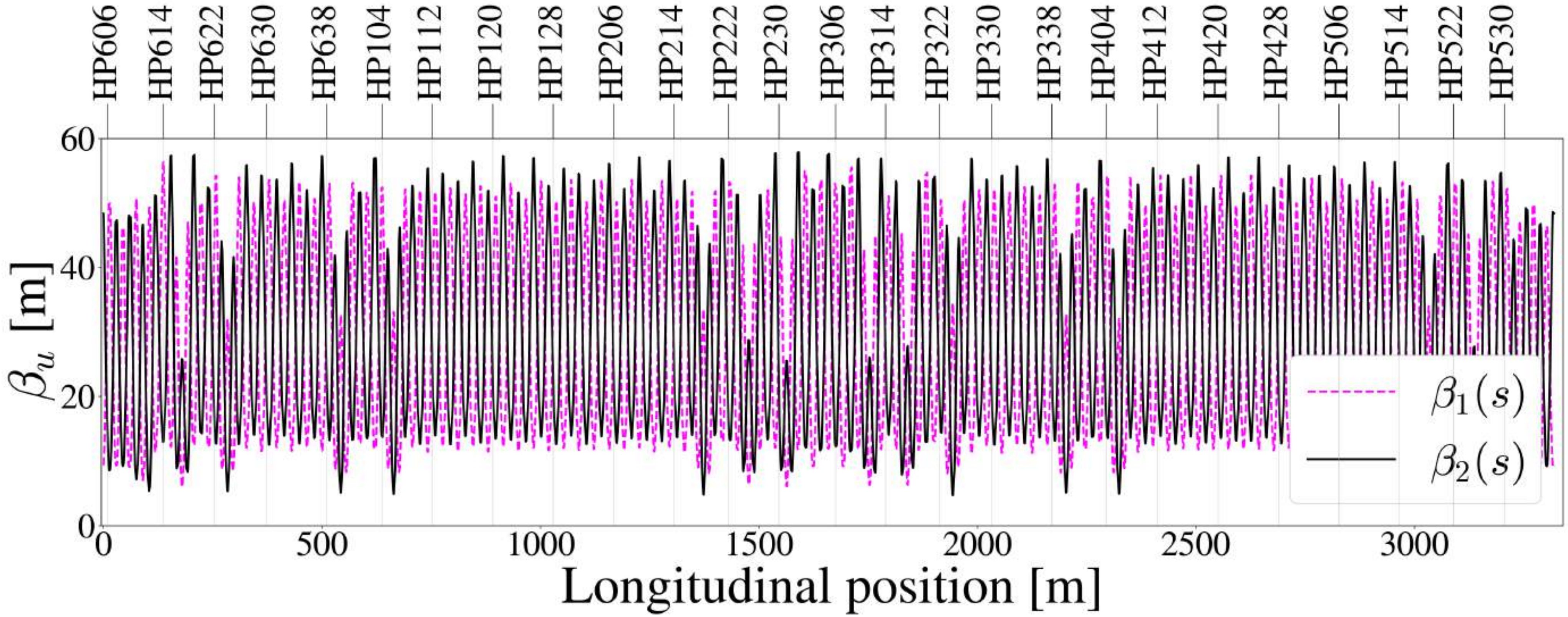
Edwards & Teng formalism formulate beta functions for coupled modes 1 and 2.

$$\beta_1, \alpha_1, \mu_1 \qquad\qquad \beta_2, \alpha_2, \mu_2$$

If there is no coupling then:

$\beta_{1y} = \beta_{2x} = 0$ $\qquad \beta_1 = \beta_{1x} = \beta_x$

$\alpha_{1y} = \alpha_{2x} = 0$ $\qquad \beta_2 = \beta_{2y} = \beta_y$



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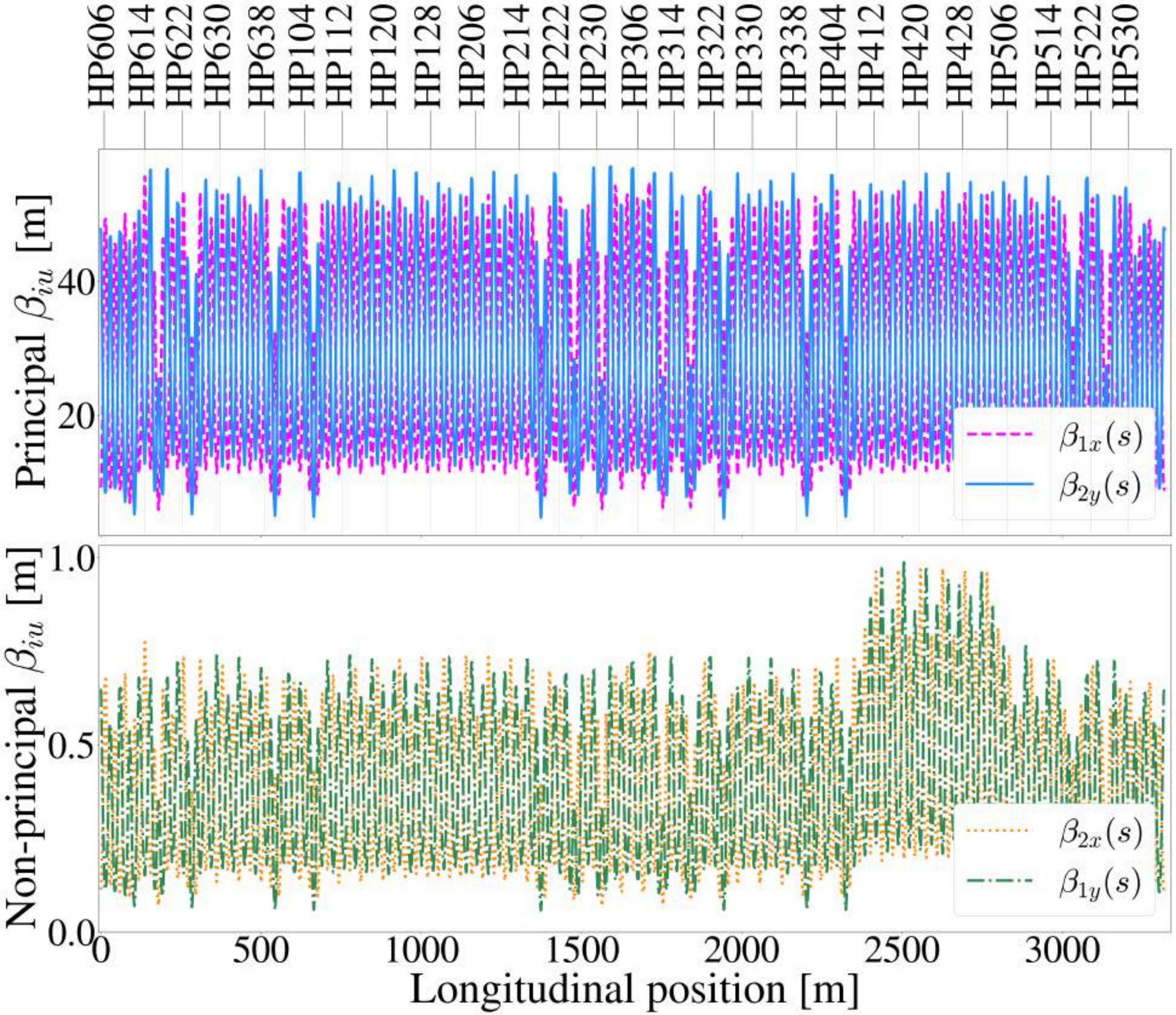
Linear Coupling Formalism: Lebedev & Bogacz

RECYCLER RING
BARE MACHINE

$$Q_x = 25.44$$
$$Q_y = 24.39$$

(MAD-X Calculation)

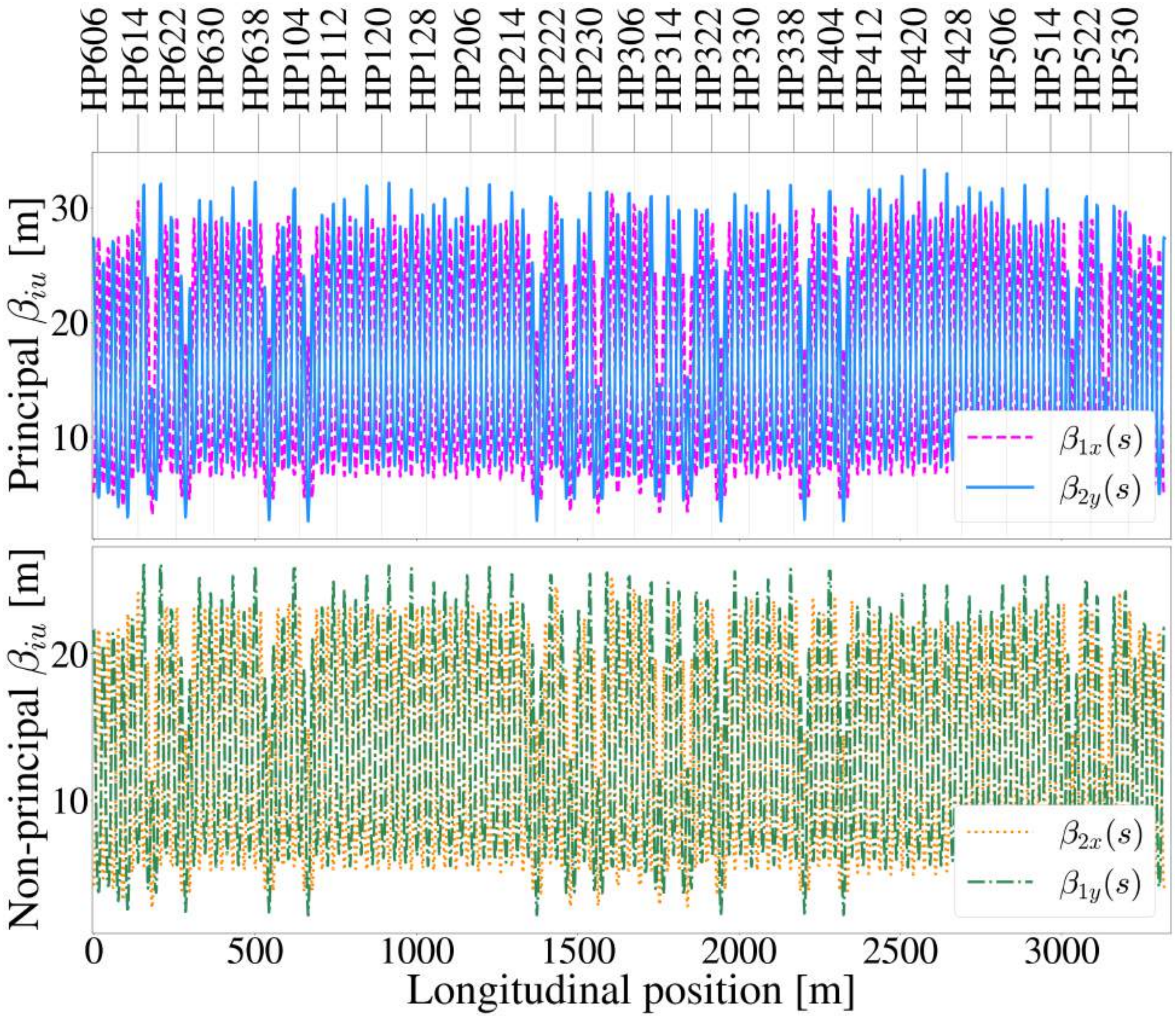


Linear Coupling Formalism: Lebedev & Bogacz

RECYCLER RING COUPLED TUNES

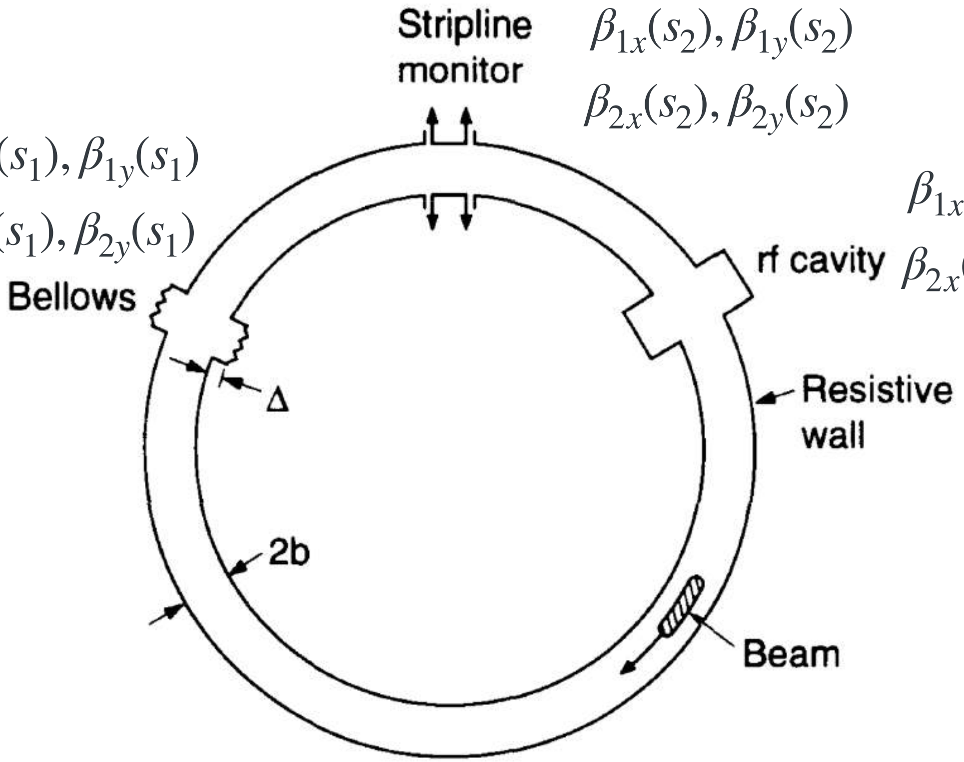
$$Q_1 = 25.396$$
$$Q_2 = 24.407$$

(MAD-X Calculation)



Single-Kick Model and Linear Coupling

We can change the single-kick model to include linear coupling using the Burov-Lebedev substitution rule. The Vlasov Equation can still be solved in terms of mode 1 and mode 2.



$\beta_{1x}(s_1), \beta_{1y}(s_1)$
$\beta_{2x}(s_1), \beta_{2y}(s_1)$

$\beta_{1x}(s_2), \beta_{1y}(s_2)$
$\beta_{2x}(s_2), \beta_{2y}(s_2)$

$\beta_{1x}(s_3), \beta_{1y}(s_3)$
$\beta_{2x}(s_3), \beta_{2y}(s_3)$

Burov-Lebedev Substitution Rule

$$Q_x \to Q_1 \qquad Z_x\beta_x \to Z_x\beta_{1x} + Z_y\beta_{1y}$$

$$Q_y \to Q_2 \qquad Z_y\beta_y \to Z_x\beta_{2x} + Z_y\beta_{2y}$$

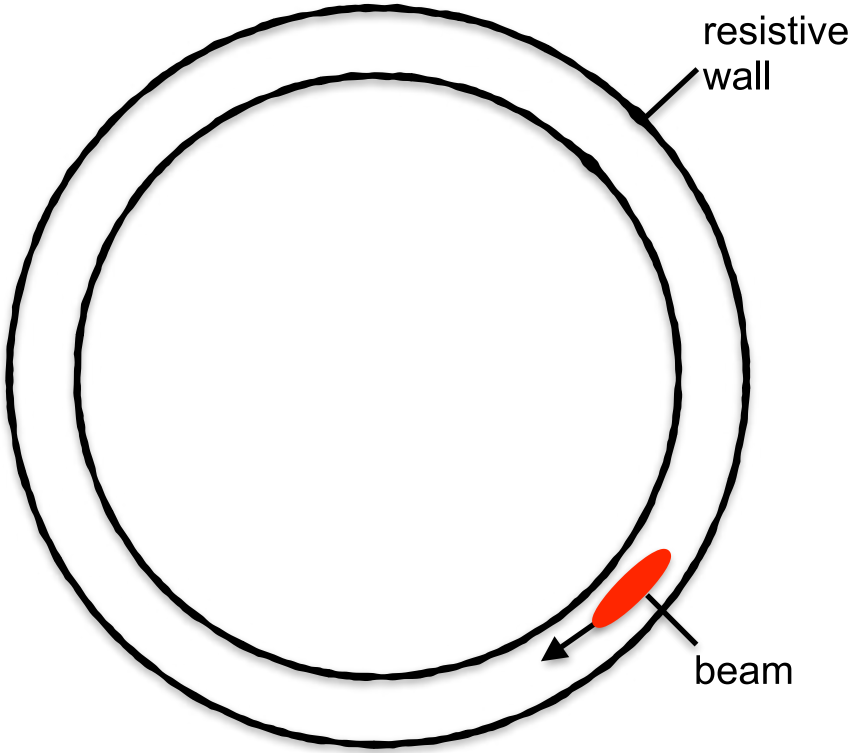
$$\langle\beta_1\rangle Z_1 = \left[\sum_j \beta_{1x}(s_j)Z_{x,j} + \beta_{1y}(s_j)Z_{y,j}\right]$$

$$\langle\beta_2\rangle Z_2 = \left[\sum_j \beta_{2x}(s_j)Z_{x,j} + \beta_{2y}(s_j)Z_{y,j}\right]$$

A. Burov and V. Lebedev, Coherent motion with linear coupling, PRAB 10, 044402 (2007).

Single-Kick Model and Linear Coupling for RW

The effective impedance of the machine for a distributed resistive wall will now be proportional to the integral of principal and non-principal beta functions around the machine, multiplied by Yokoya factors.



$$\langle\beta_1\rangle Z_1(\omega) = R_W(\omega)\left[G_{1x}\left(\frac{1}{C}\int_0^C \beta_{1x}(s)\,ds\right) + G_{1y}\left(\frac{1}{C}\int_0^C \beta_{1y}(s)\,ds\right)\right]$$

$$\langle\beta_2\rangle Z_2(\omega) = R_W(\omega)\left[G_{1x}\left(\frac{1}{C}\int_0^C \beta_{2x}(s)\,ds\right) + G_{1y}\left(\frac{1}{C}\int_0^C \beta_{2y}(s)\,ds\right)\right]$$

Effective impedance will be proportional to optical-geometrical factors F_1 and F_2

$$F_1 \equiv G_{1x}\left(\frac{1}{C}\int_0^C \beta_{1x}(s)\,ds\right) + G_{1y}\left(\frac{1}{C}\int_0^C \beta_{1y}(s)\,ds\right)$$

$$F_2 \equiv G_{1x}\left(\frac{1}{C}\int_0^C \beta_{2x}(s)\,ds\right) + G_{1y}\left(\frac{1}{C}\int_0^C \beta_{2y}(s)\,ds\right)$$

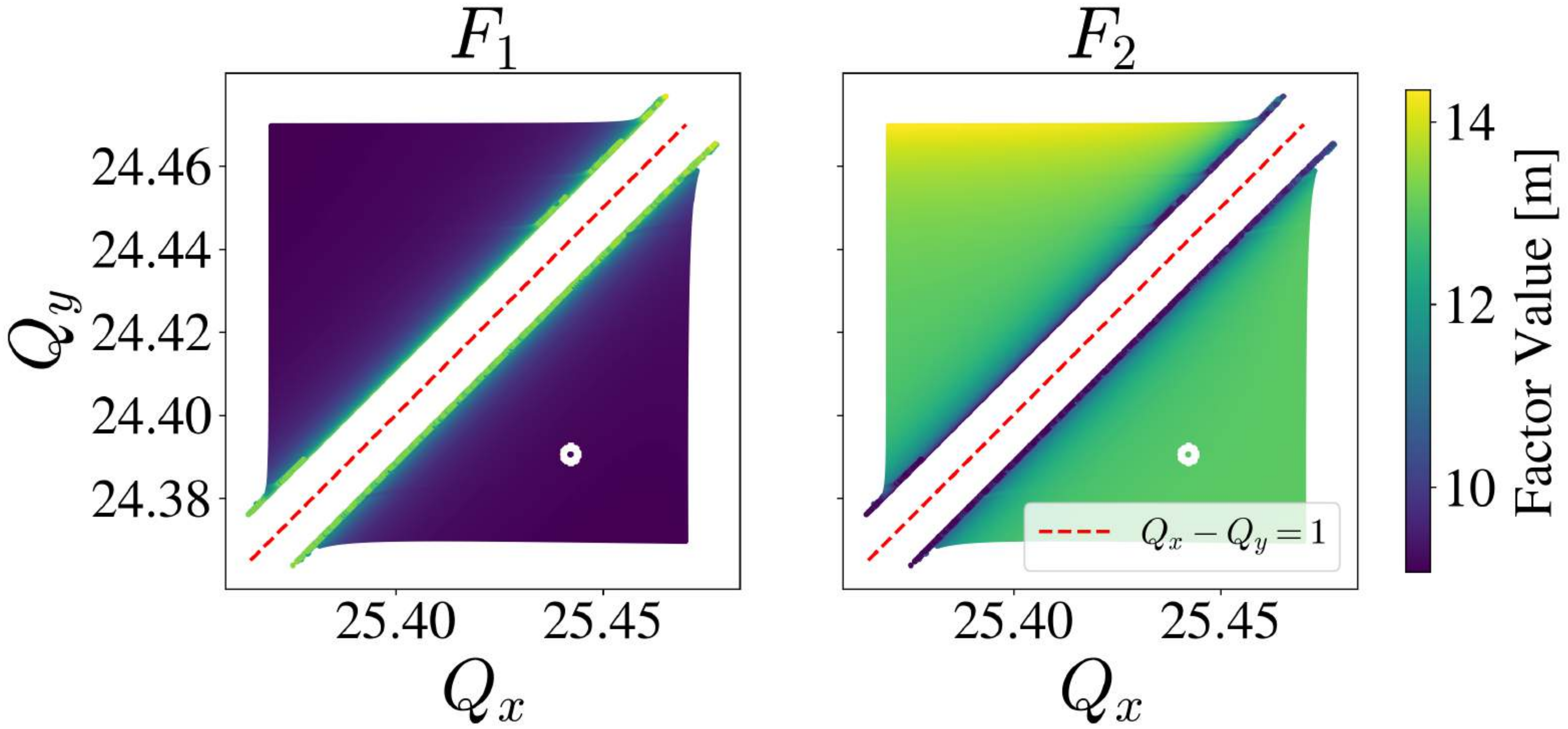
Yokoya factors for elliptical beam pipe

$G_{1x} = 0.4501$

$G_{1y} = 0.8369$

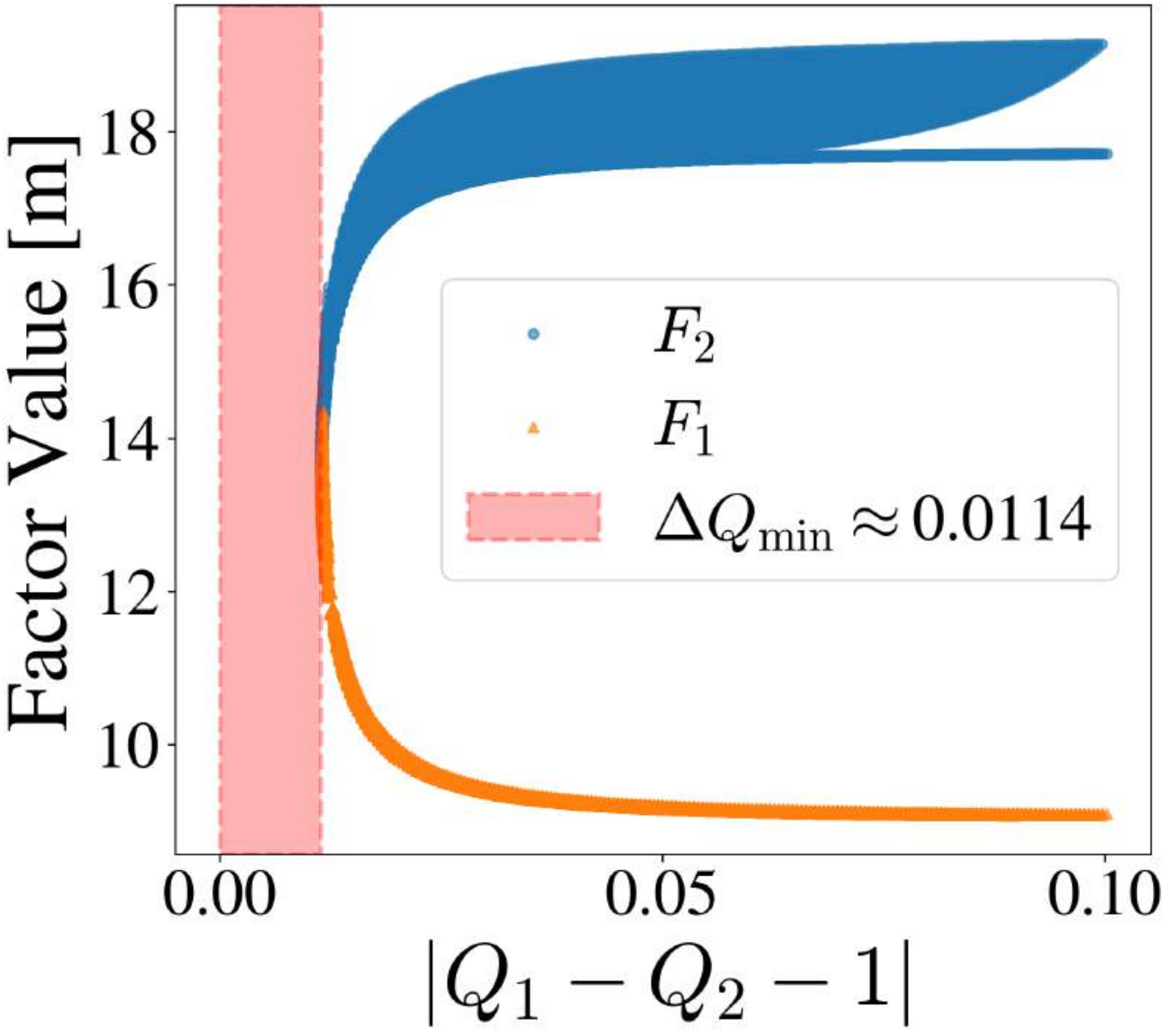
Single-Kick Model and Linear Coupling: Optical-Geometrical F

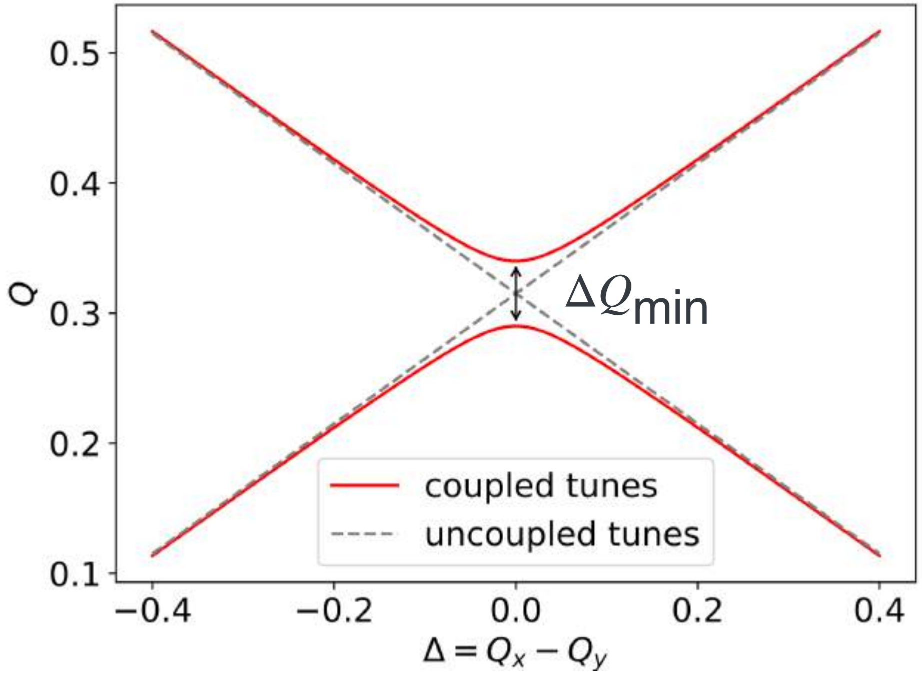
The optical geometrical factors F_1 and F_2 can be calculated for several tunes of the machine. Nevertheless, due to coupling there is a fringe which you can never tune Q_1 and Q_2 to.



Single-Kick Model and Linear Coupling

Without coupling the fractional tunes Q_x and Q_y can be matched to the same value. When coupling is present the two coupled fractional tunes Q_1 and Q_2 cannot be matched to the same value, but must have a minimal separation of ΔQ_{min}.

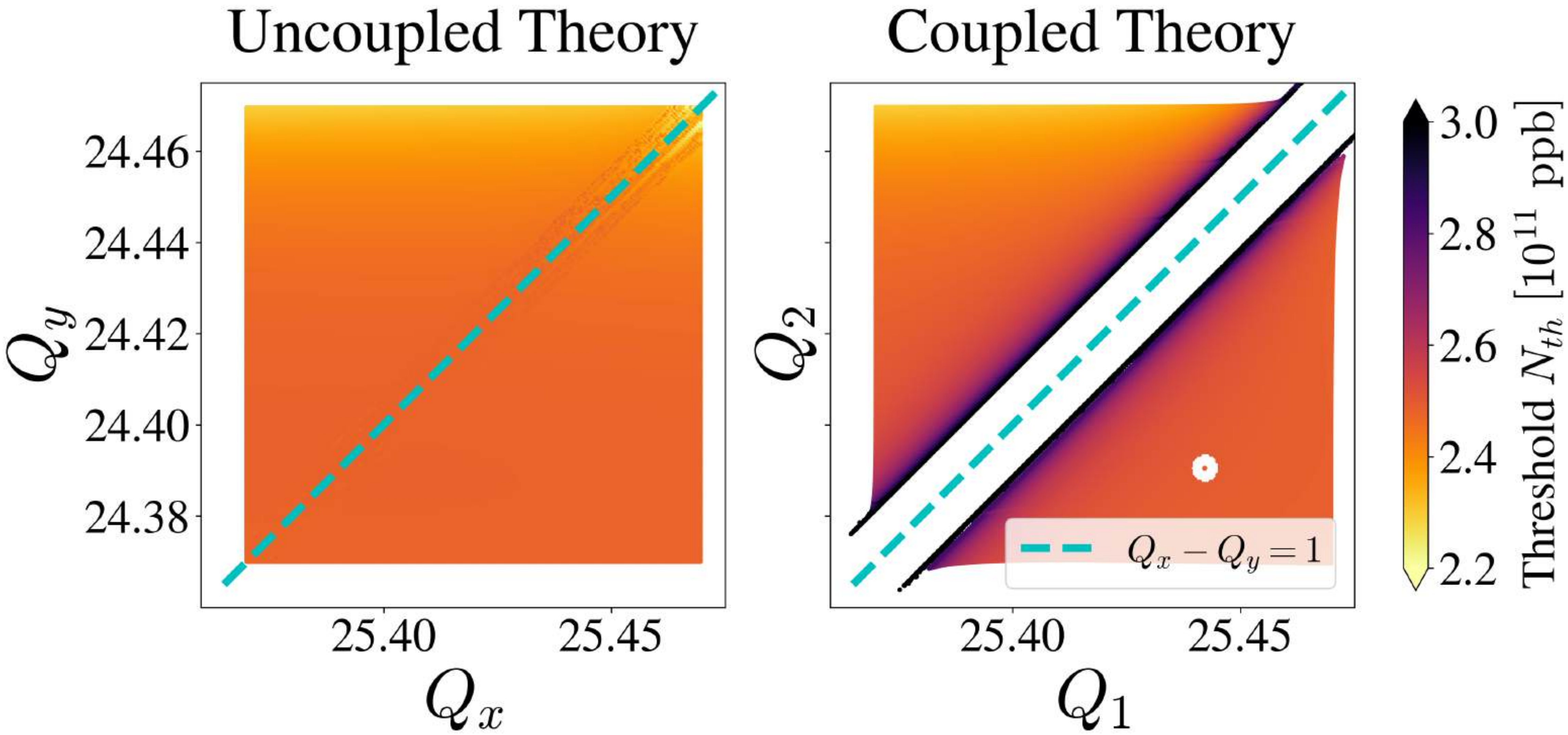




R. Tomas, E. J. Høydalsvik, T. Persson, Evaluation of the closest tune approach and its MAD-X implementation, Tech. Rep. CERN-ACC-NOTE-2021-0022

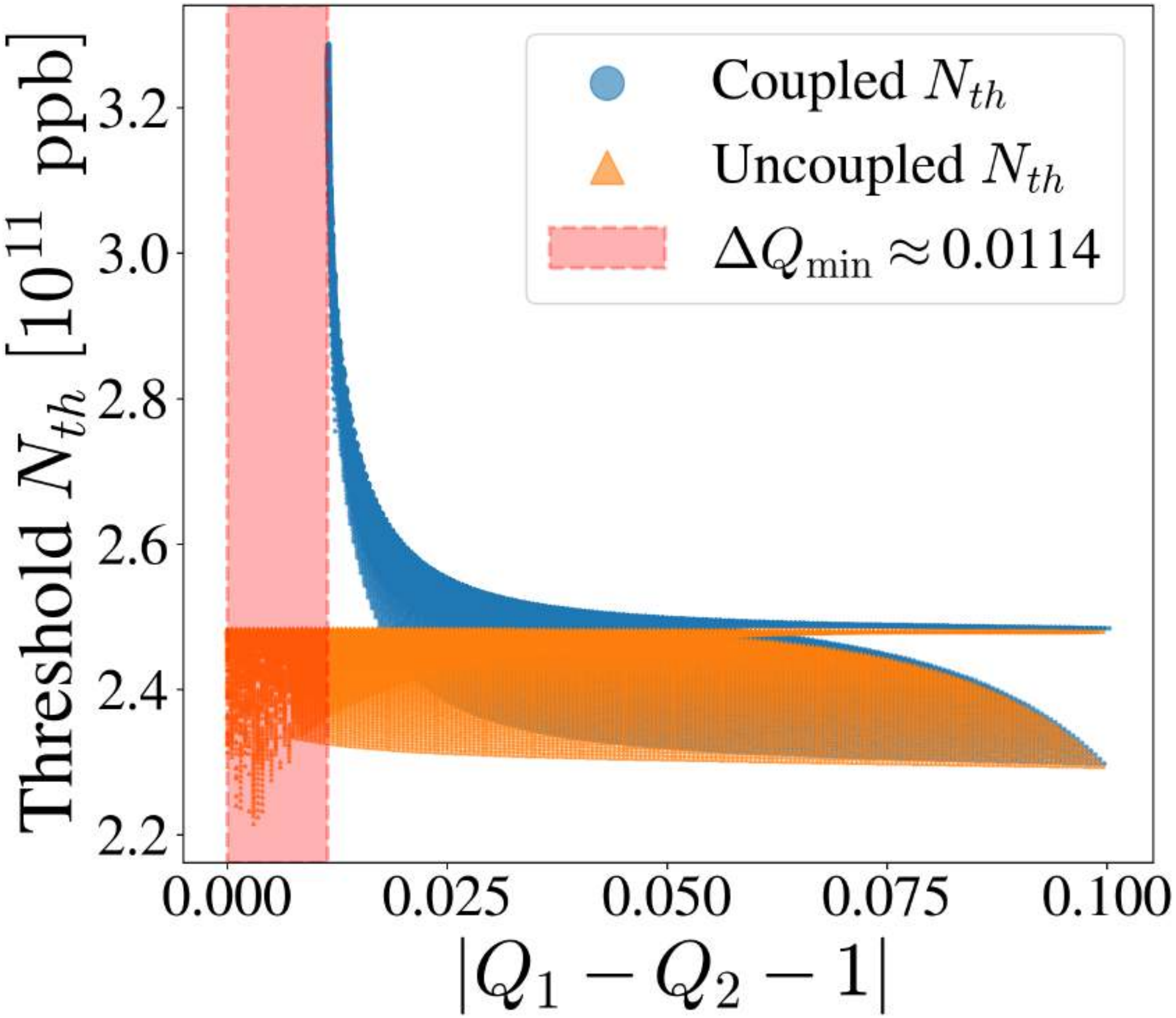
TMCI Thresholds and Linear Coupling

With the effective impedance and optical-geometrical factors F_1 and F_2 one can use the NHT model to estimate TMCI thresholds at different tunes.



TMCI Thresholds and Linear Coupling

The thresholds including linear coupling tell us the opposite of what we see experimentally. Our model predicts that at the coupling line, the threshold should go up. We see the opposite.



Just including linear coupling to the single-kick model does not explain our experimental data.

The next most likely contributor is space charge, which modifies the coherent tune spectrum, alters mode coupling conditions, and can significantly reshape the instability thresholds.

05

Theory & Simulations (Part 3: Space Charge)

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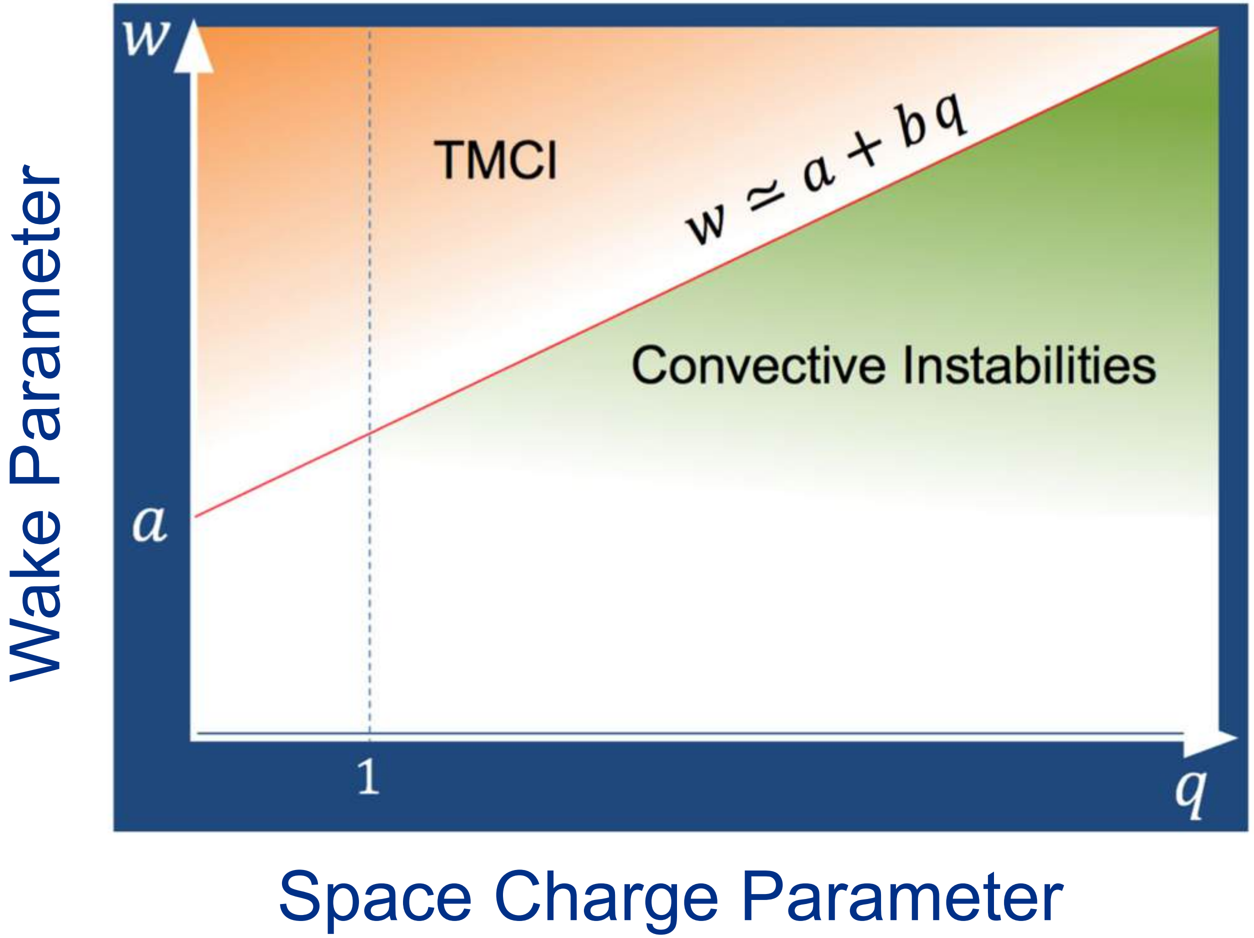
TMCI & Space Charge: Convective Instabilities

TMCI threshold changes with space charge parameter. Nevertheless, at high space charge we have to deal with convective instabilities.



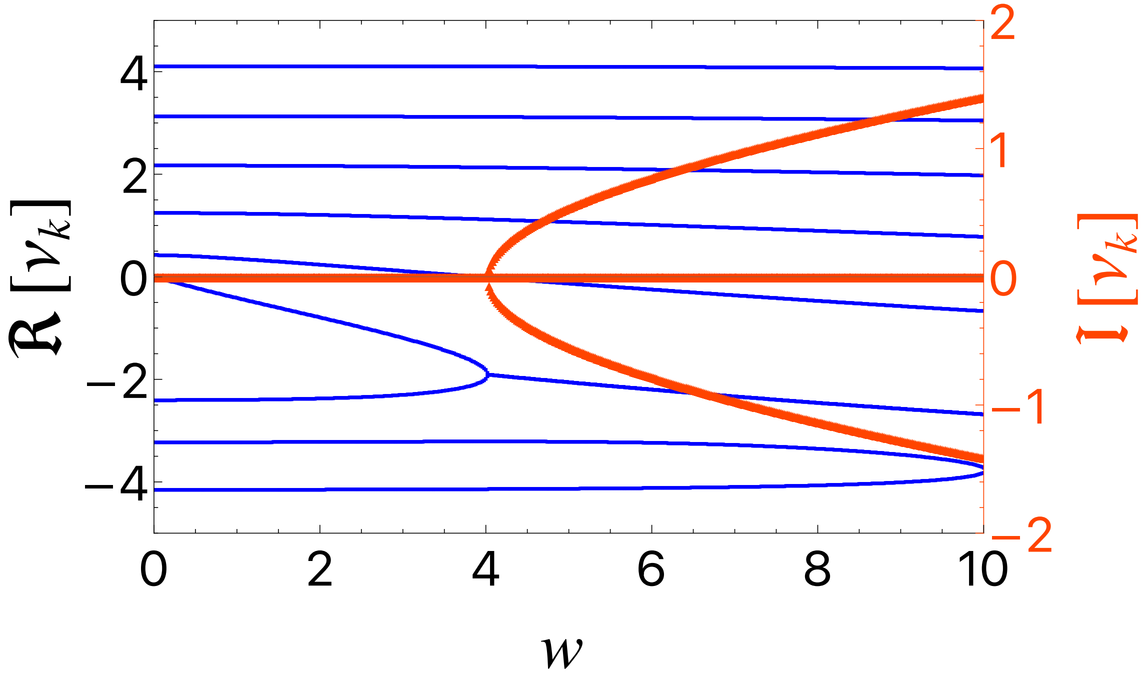
The TMCI threshold depends on the space charge parameter. This is the space charge tune shift (incoherent) divided by the synchrotron tune.

$$q_{SC} = \frac{\Delta Q_{\beta,i}}{Q_s}$$

A. Burov, "Convective instabilities of bunched beams with space charge," PRAB, vol. 22, no. 3, p. 034202, 2019

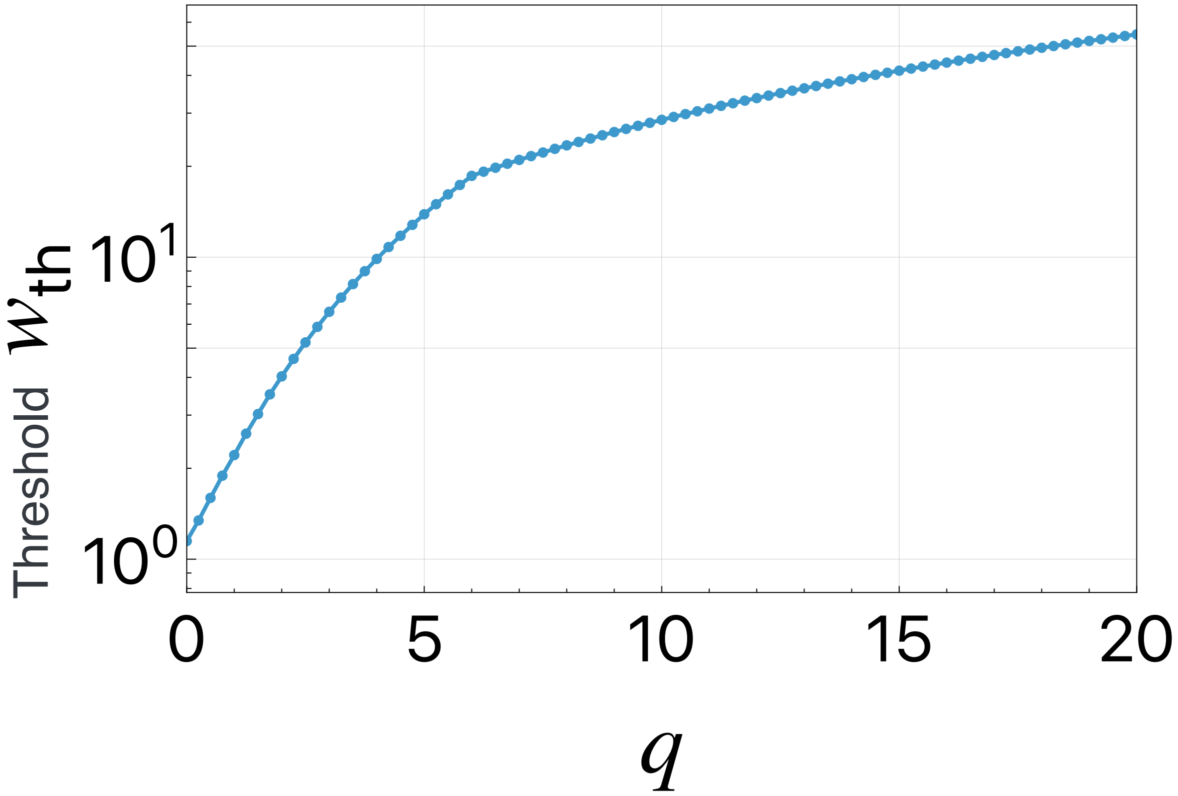
TMCI & Space Charge: Convective Instabilities

Initial calculations with a simple theta (constant) wake for the airbag in a square well model (ABS).
The TMCI threshold increases when space charge parameter is increased.



Intensity/Wake parameter

A. Burov, "Convective instabilities of bunched beams with space charge," PRAB, vol. 22, no. 3, p. 034202, 2019

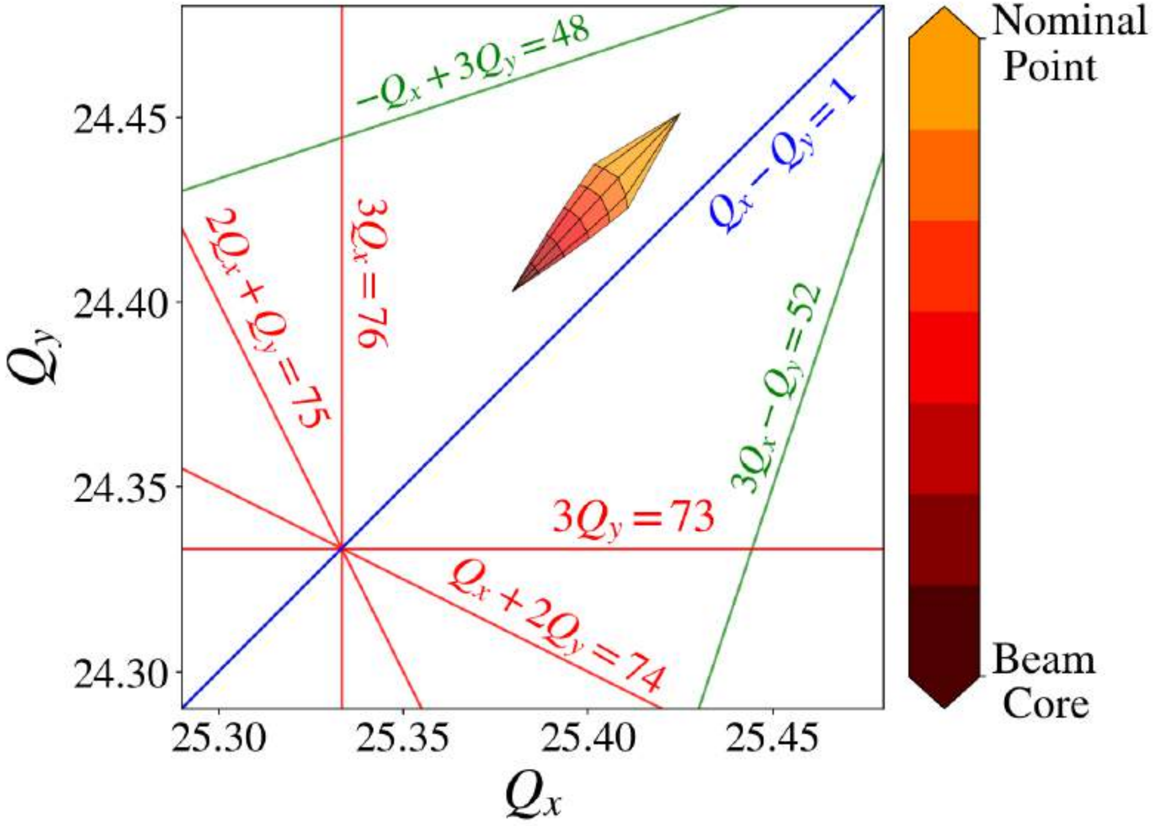


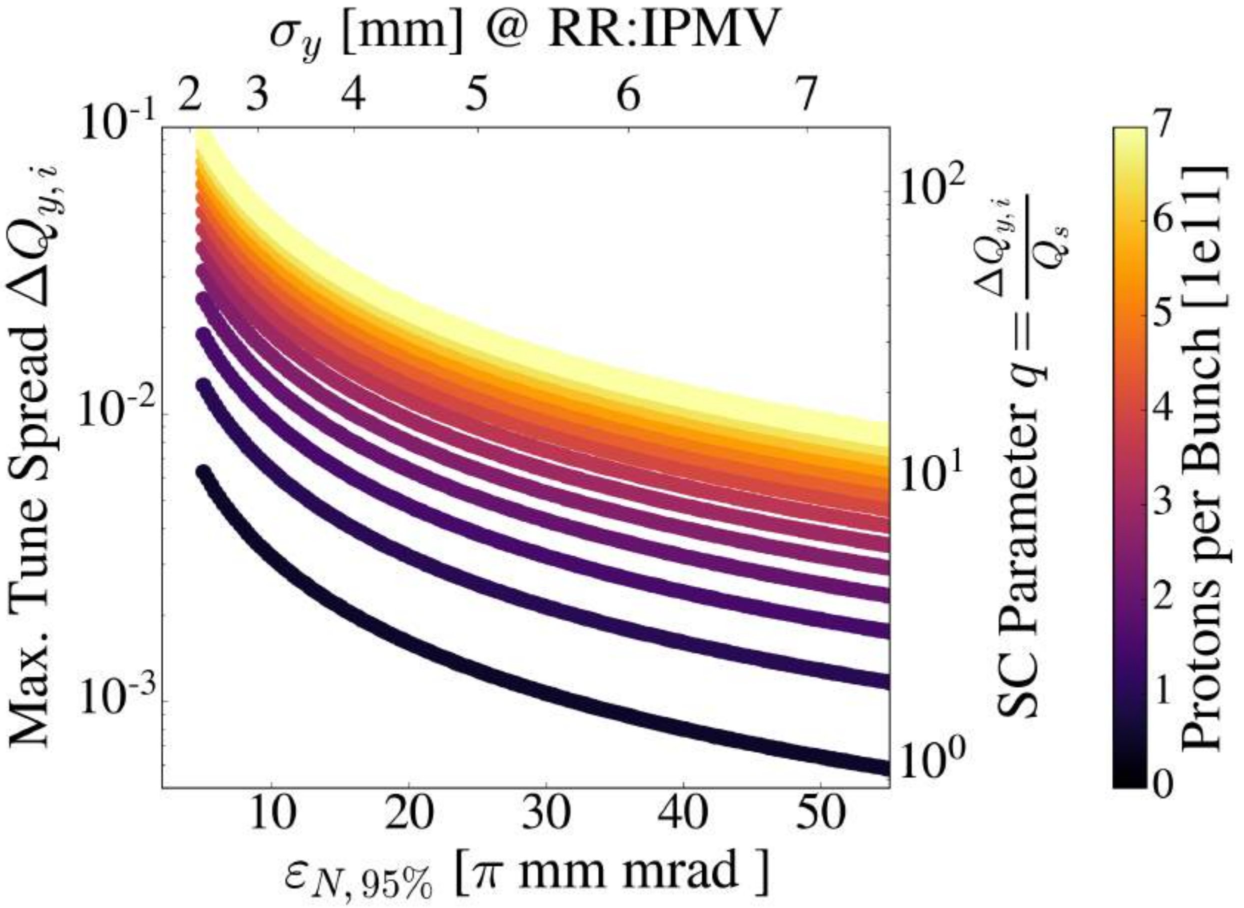
Space Charge Parameter

M. Blaskiewicz, Fast Head Tail Instability with Space Charge. PRAB 1, 044201(1998).

Space Charge Parameters in the Recycler Ring

One can calculate the space charge tune shifts for uncoupled modes $\Delta Q_{sc,x}$ and $\Delta Q_{sc,y}$ for a given intensity and a given normalized emittance.

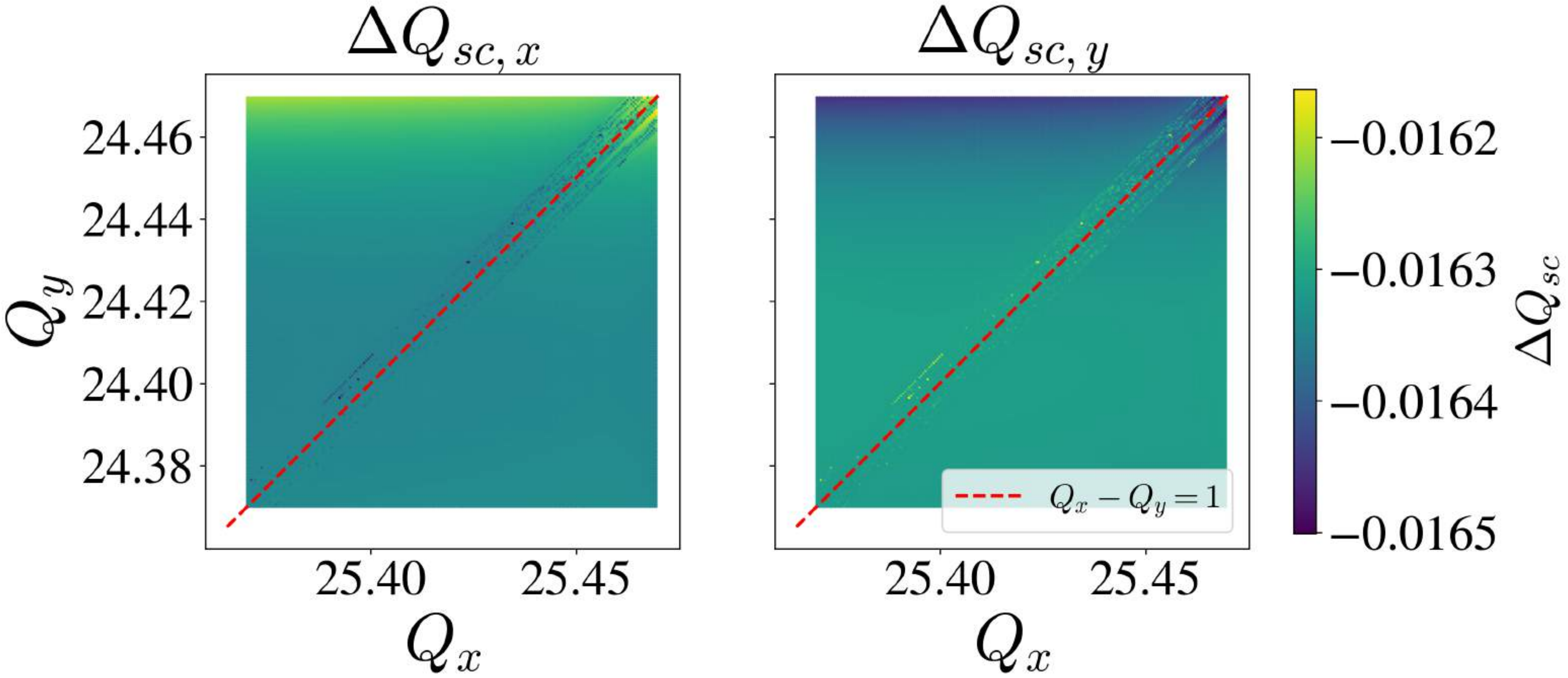




Space Charge Parameters in the Recycler Ring

One can calculate the space charge tune shifts for uncoupled modes $\Delta Q_{sc,x}$ and $\Delta Q_{sc,y}$, nevertheless this doesn't take into account linear coupling.

Intensity: 4×10^{11} protons per bunch



Our current hypothesis is that the space-charge tune shifts of the coupled modes, $\Delta Q_{sc,1}$ and $\Delta Q_{sc,2}$, decrease as the working point approaches the coupling resonance, thereby reducing the TMCI threshold.

We still need to compute the space-charge tune shifts for the linearly coupled modes—a nontrivial task, as the final explanation is likely to involve the combined effects of linear coupling and space charge.

Conclusions & Future Work

Linear coupling affects the effective impedance of circular accelerators.

Linear coupling modifies how each coupled eigenmode samples the horizontal and vertical wakefields, redistributing the impedance between them.

Semi-analytical models (NHT model) can help accelerate calculations of TMCI thresholds.

These models capture the essential beam-impedance dynamics using a reduced basis of head–tail modes, enabling fast and accurate stability predictions.

Further theoretical and experimental work should be on space charge and single bunch instabilities.

Future work work should focus on the interplay between space charge and single-bunch instabilities. The WAKER experiment will help.

THANK YOU! ANY QUESTIONS?