OAK RIDGE National Laboratory

Eigenmode painting in high-intensity accelerators

Austin Hoover (with Nick Evans, Vasiliy Morozov, and Kelly Anderson)

APES Seminar | Michigan State University

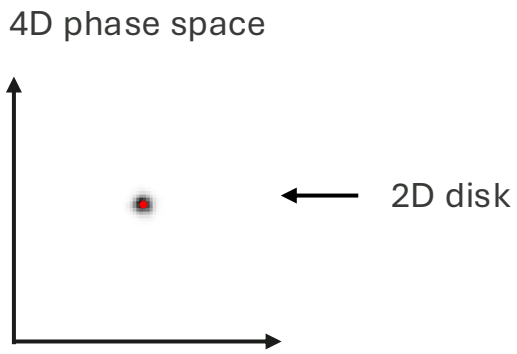
November 06, 2026

U.S. DEPARTMENT of ENERGY

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Eigenpainting is a method to generate intense beam distributions with small 4D emittance

- Method proposed by V. Danilov in 2003.
- Experimental program at SNS going since 2018.
- I will explain the method and initial experiments + planned improvements and open questions.



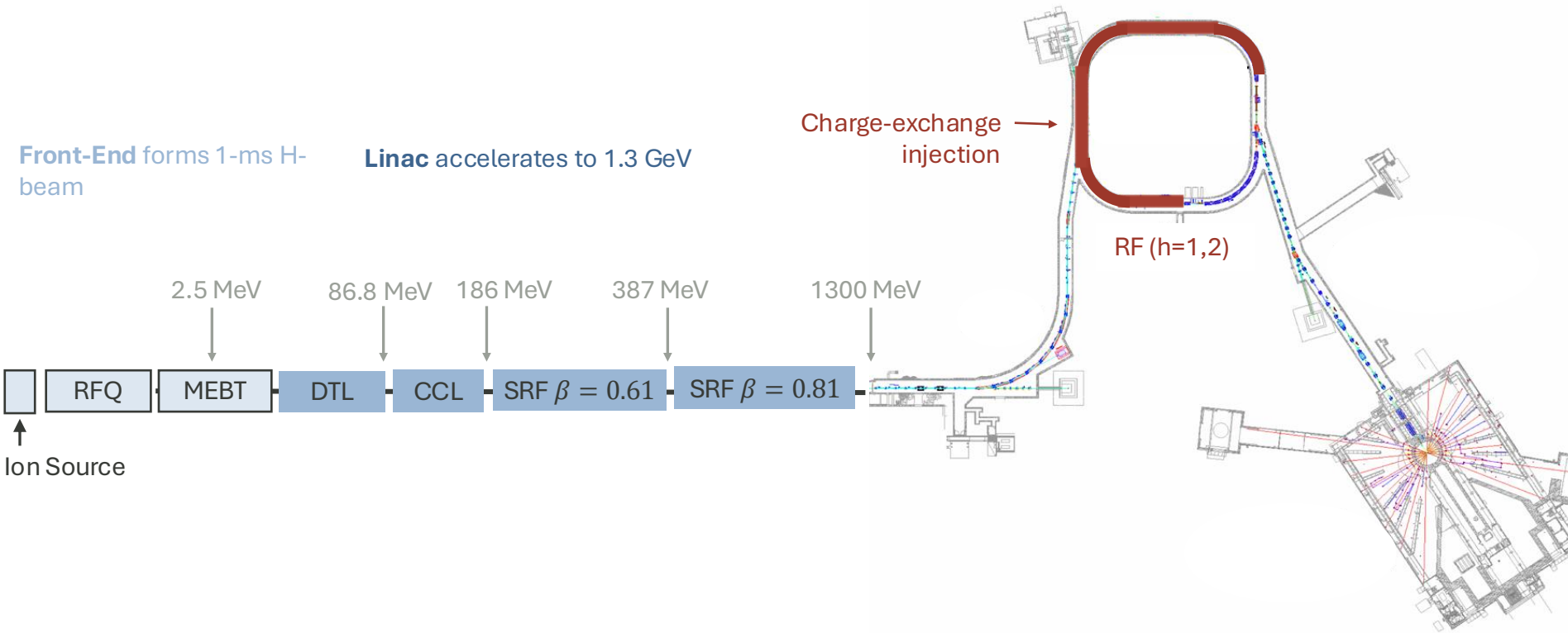
Phase space painting

SNS accumulates intense proton beams for neutron scattering applications

Front-End forms 1-ms H- beam

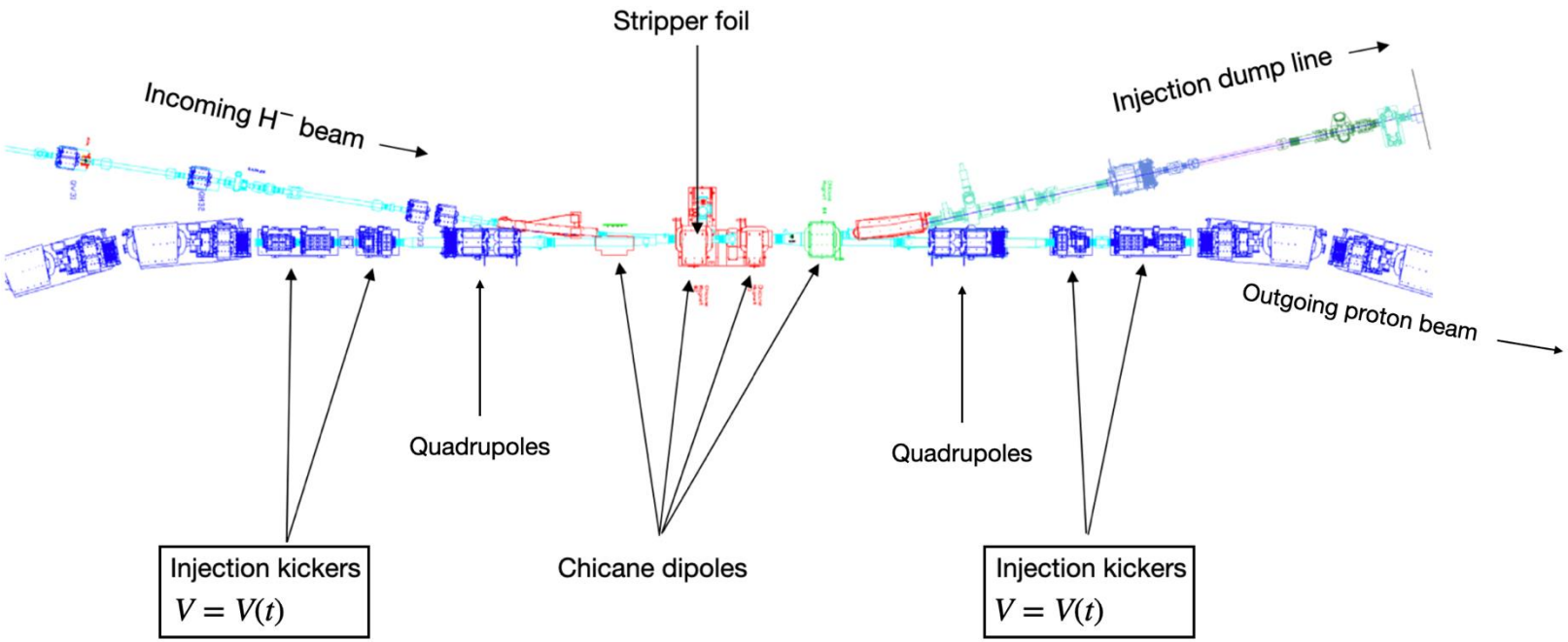
Linac accelerates to 1.3 GeV

Accumulator ring compresses to 700 ns



Target 2.0 MW @ 60 Hz

Eight time-dependent kicker (dipole) magnets control beam trajectory in injection region



Kickers determine position and angle of injected beam centroid (relative to closed orbit)

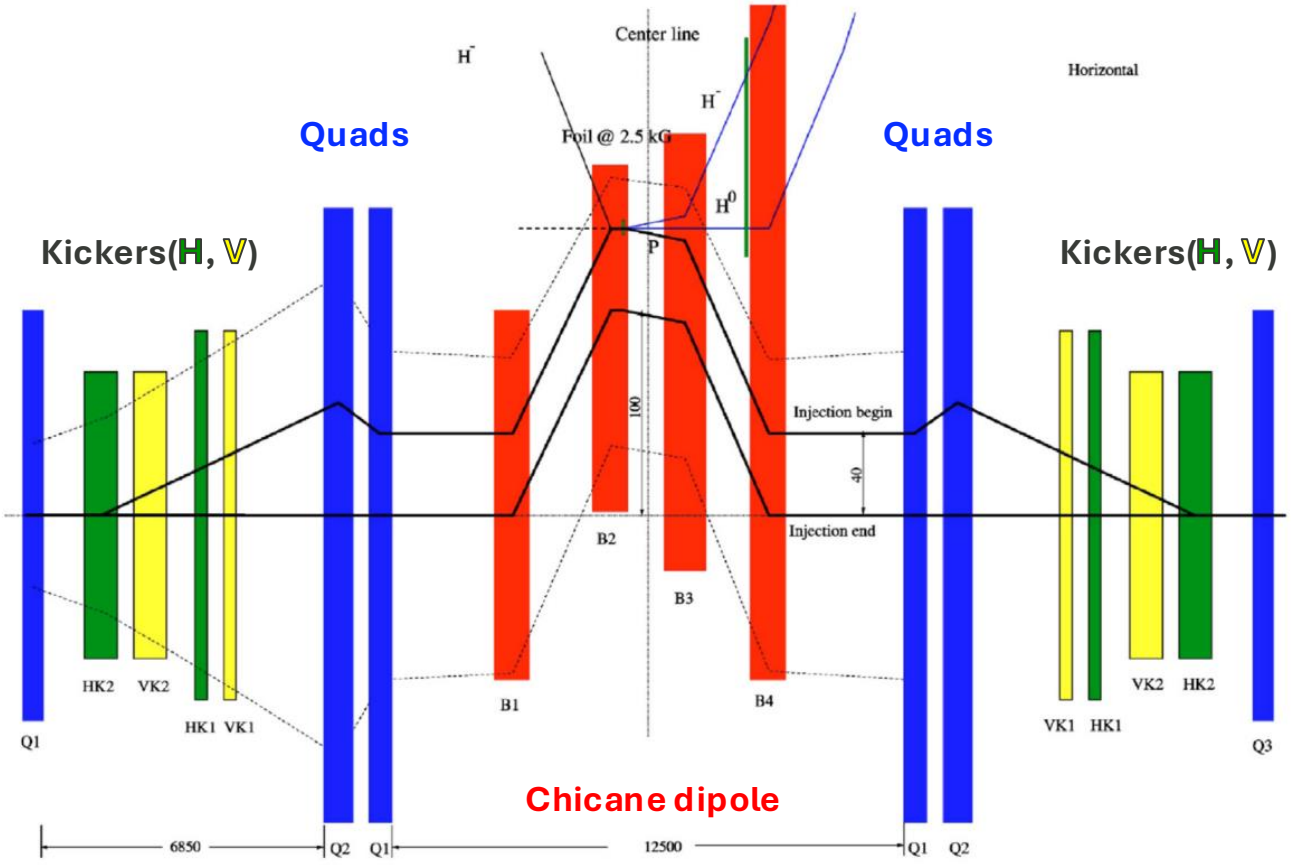
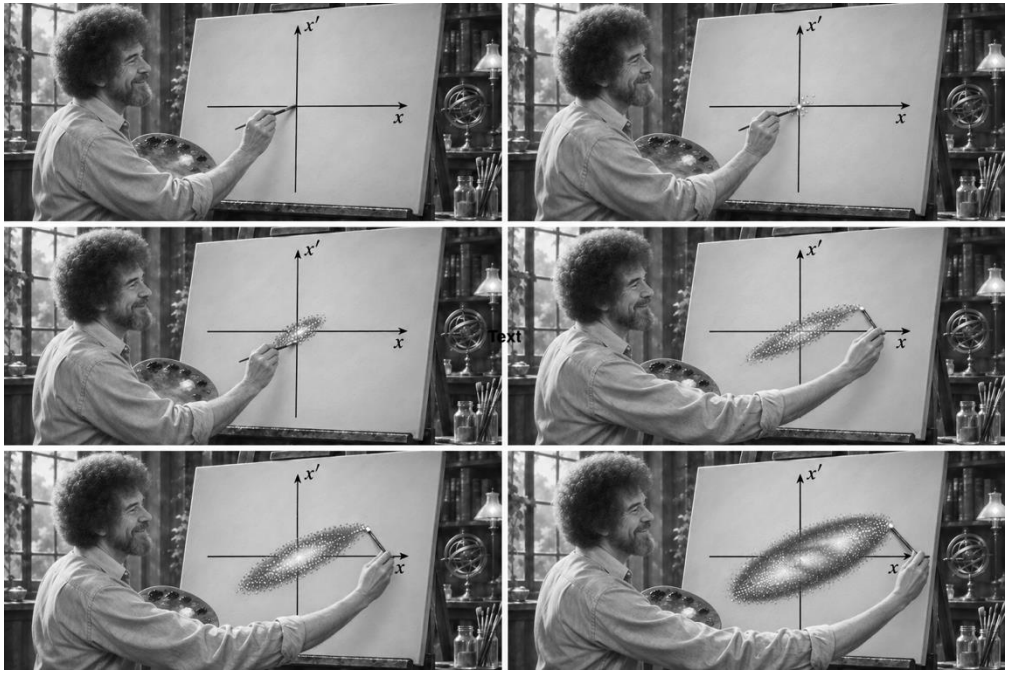


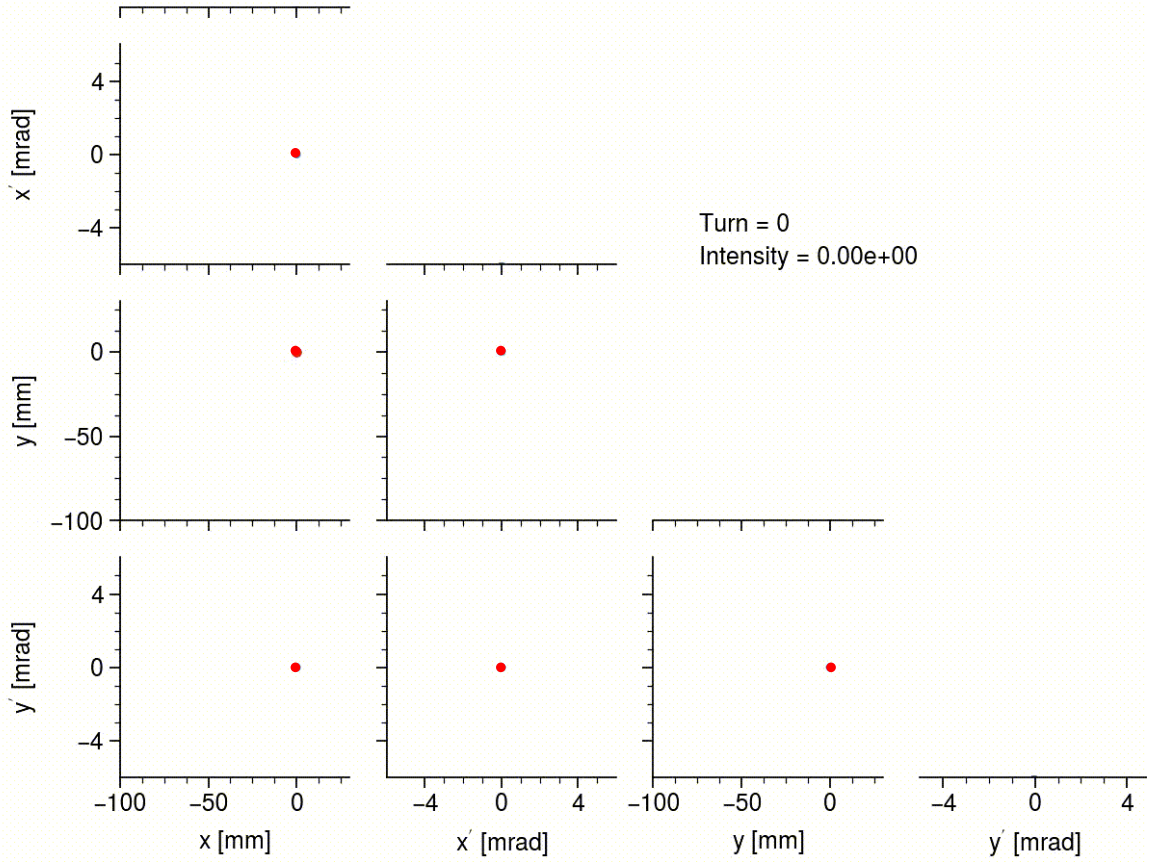
Fig. 87. Schematic layout of the horizontal plane of the beam injection region of the accumulator ring. Reprinted Figure with permission from Ref. [89] (http://link.aps.org/abstract/RMP/v75/p1383). Copyright 2003 by the American Physical Society.

Painting = variation of injection point in phase space as a function of time

$$\mathbf{x}_{inj}(t) = [x(t), x'(t), y(t), y'(t)]$$



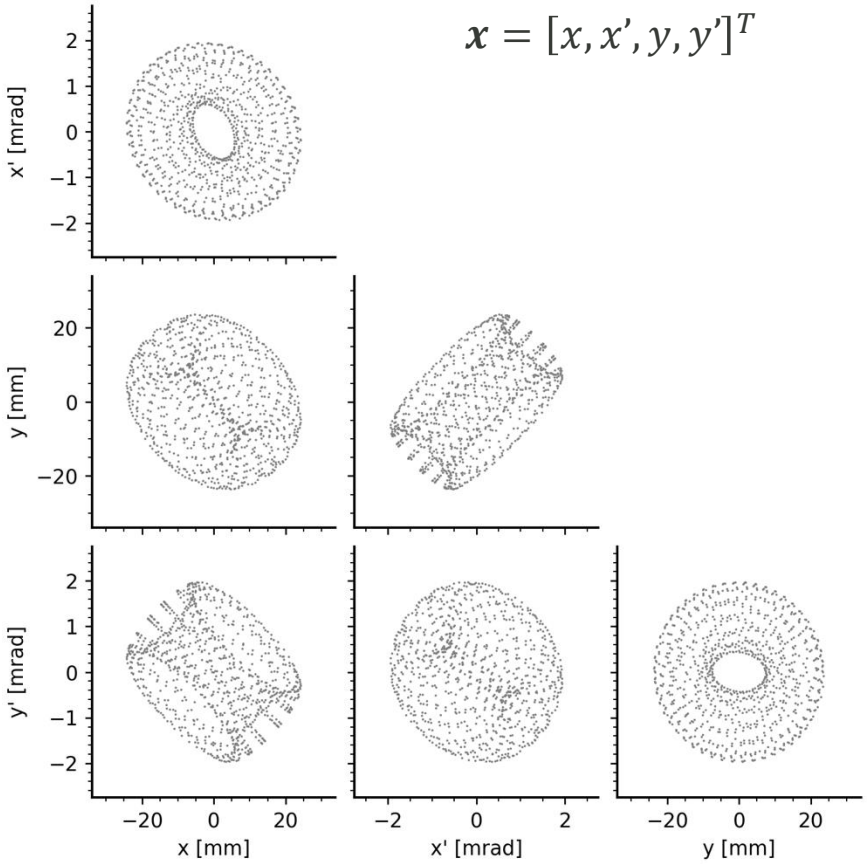
Our goal is to use painting to control the phase space distribution with space charge



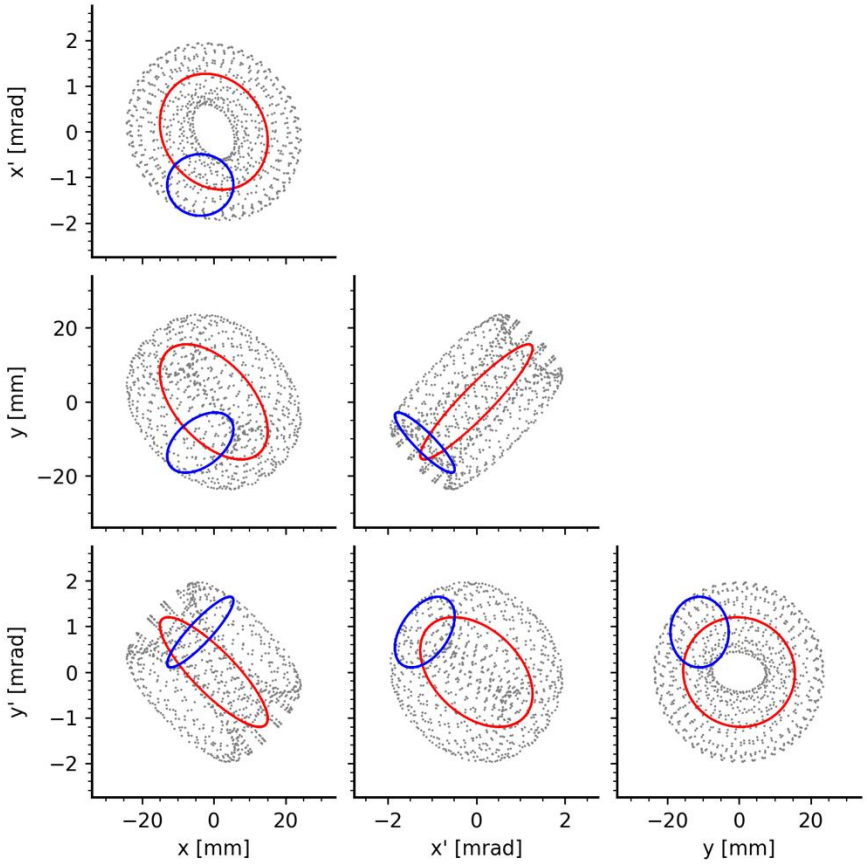
Eigenpainting

Start with turn-by-turn coordinates in linear lattice

$$\boldsymbol{x} = [x, x', y, y']^T$$



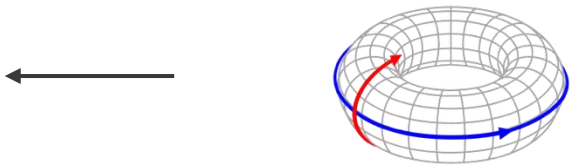
Express coordinates as linear combination of transfer matrix eigenvectors (modes)



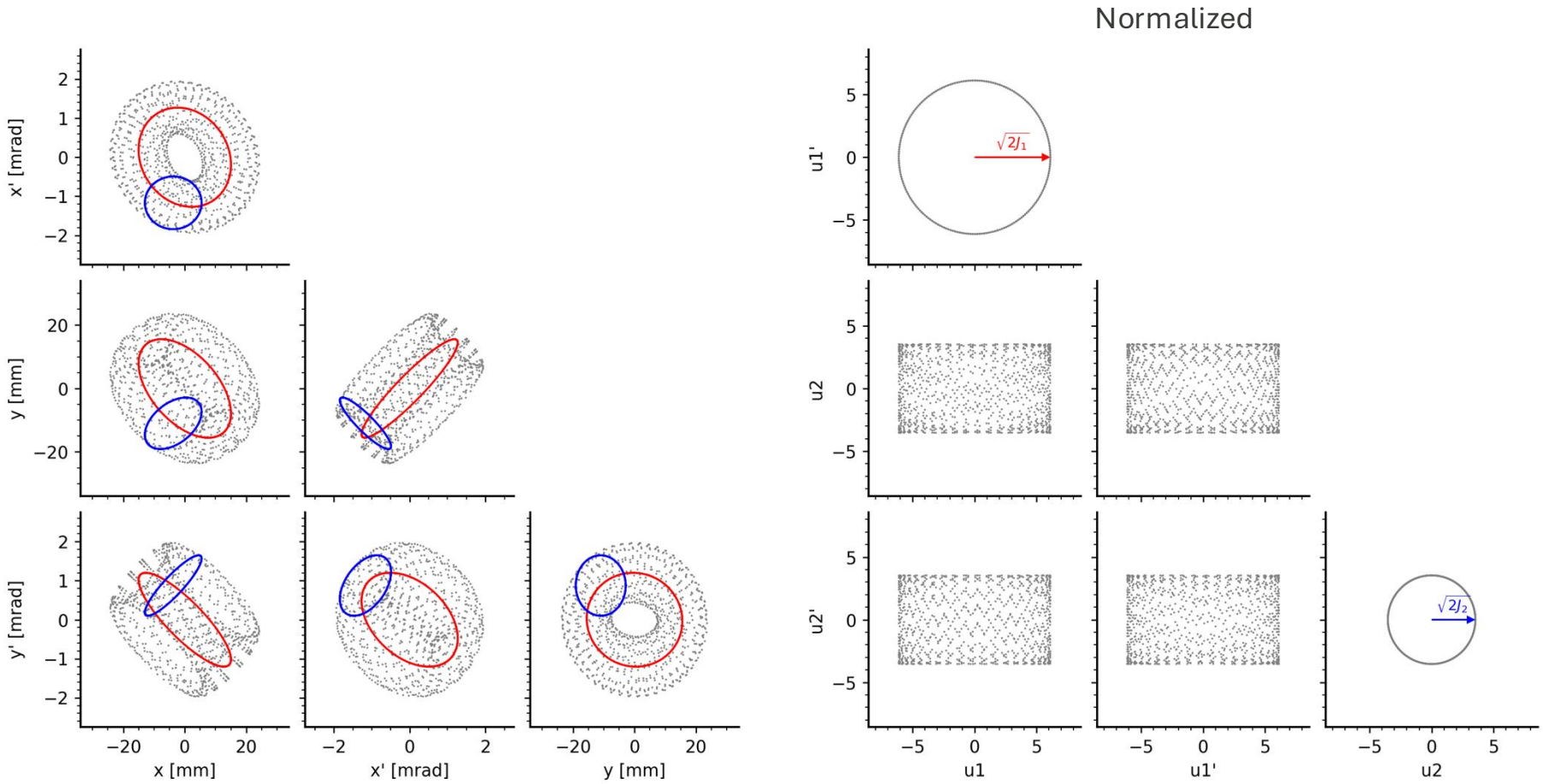
$$\boldsymbol{x} = \Re\{\sqrt{2J_1}\boldsymbol{v}_1 e^{-i\psi_1} + \sqrt{2J_2}\boldsymbol{v}_2 e^{-i\psi_2}\}$$

$$\boldsymbol{Mx} = \Re\{\sqrt{2J_1}\boldsymbol{v}_1 e^{-i(\psi_1+\mu_1)} + \sqrt{2J_2}\boldsymbol{v}_2 e^{-i(\psi_2+\mu_2)}\}$$

4D torus



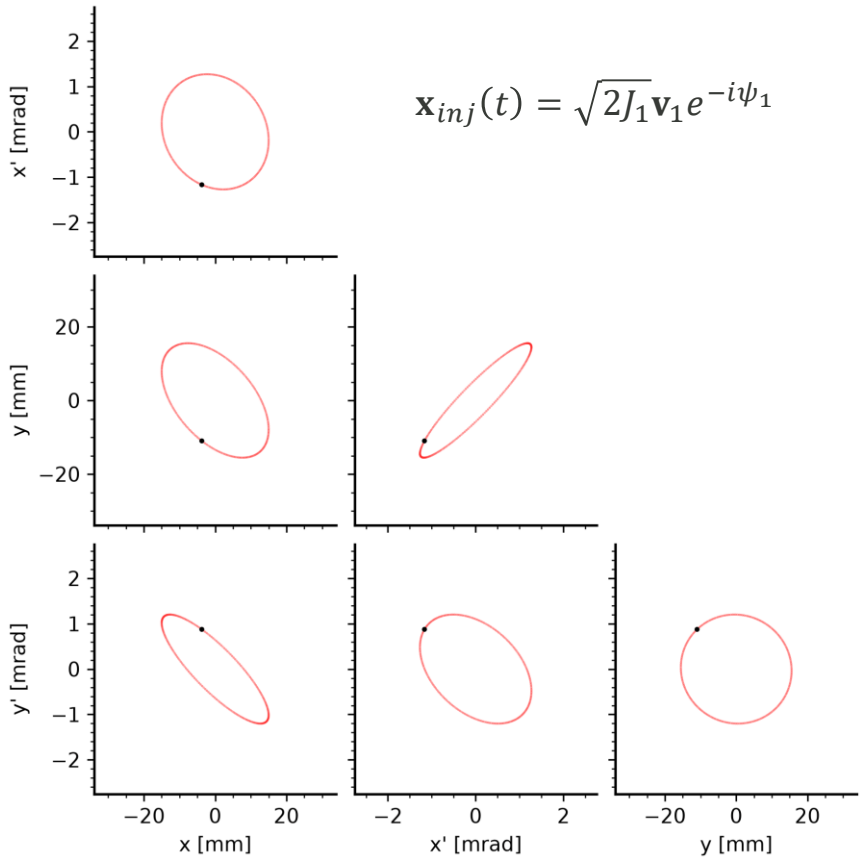
Define normalized coordinates through symplectic decoupling transformation



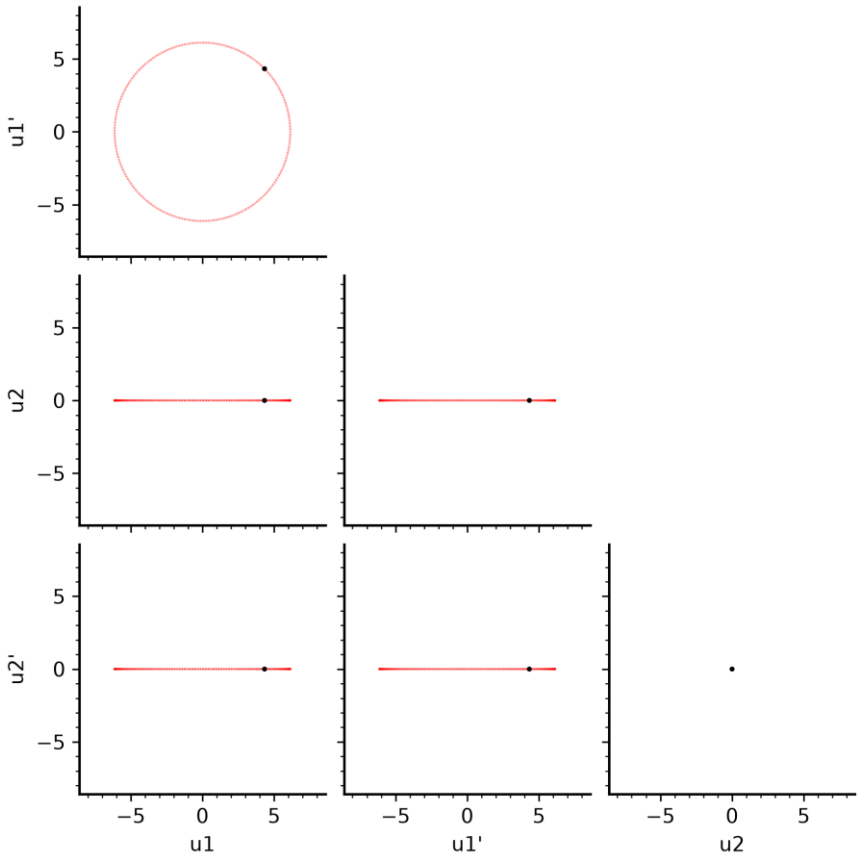
$$\mathbf{V} = [\Re\{\mathbf{v}_1\}, -\Im\{\mathbf{v}_1\}, \Re\{\mathbf{v}_2\}, -\Im\{\mathbf{v}_2\}]$$

Create elliptical paintbrush by injecting particles into one mode

$$\mathbf{x}_{inj}(t) = \sqrt{2J_1}\mathbf{v}_1 e^{-i\psi_1}$$

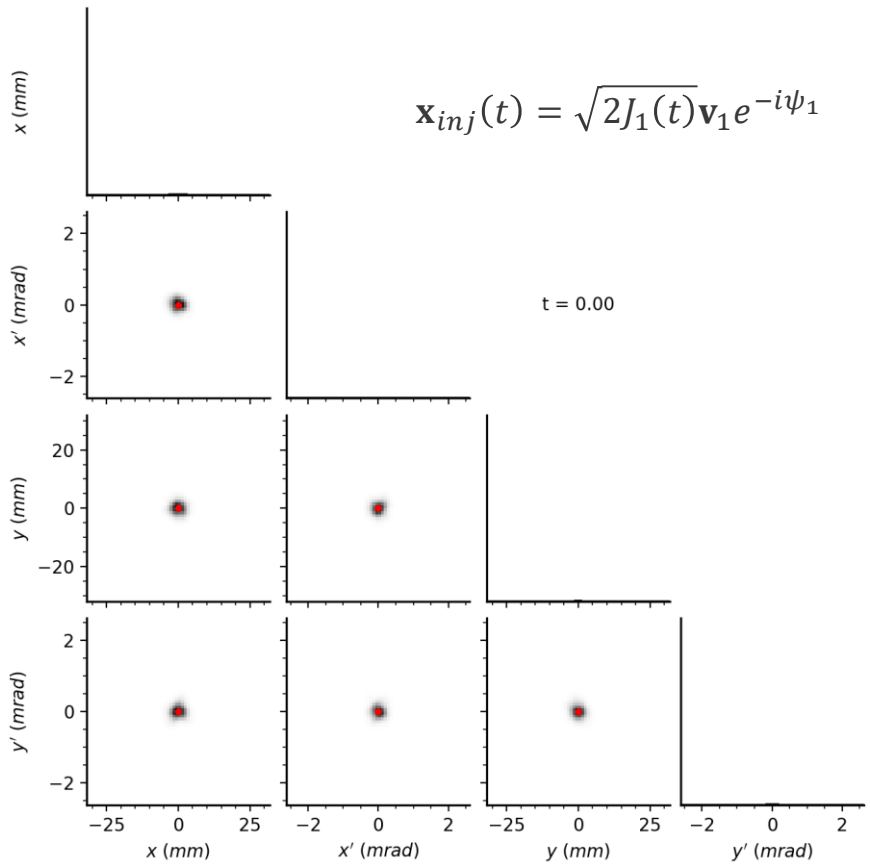


Normalized

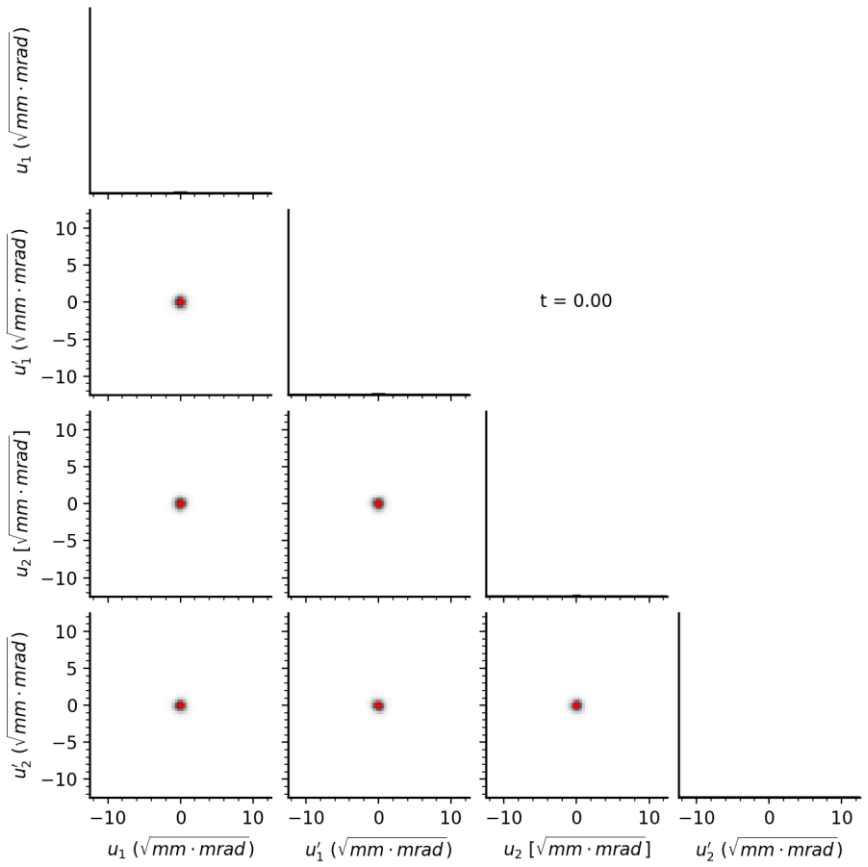


Scale injection amplitude to create desired elliptically symmetric distribution

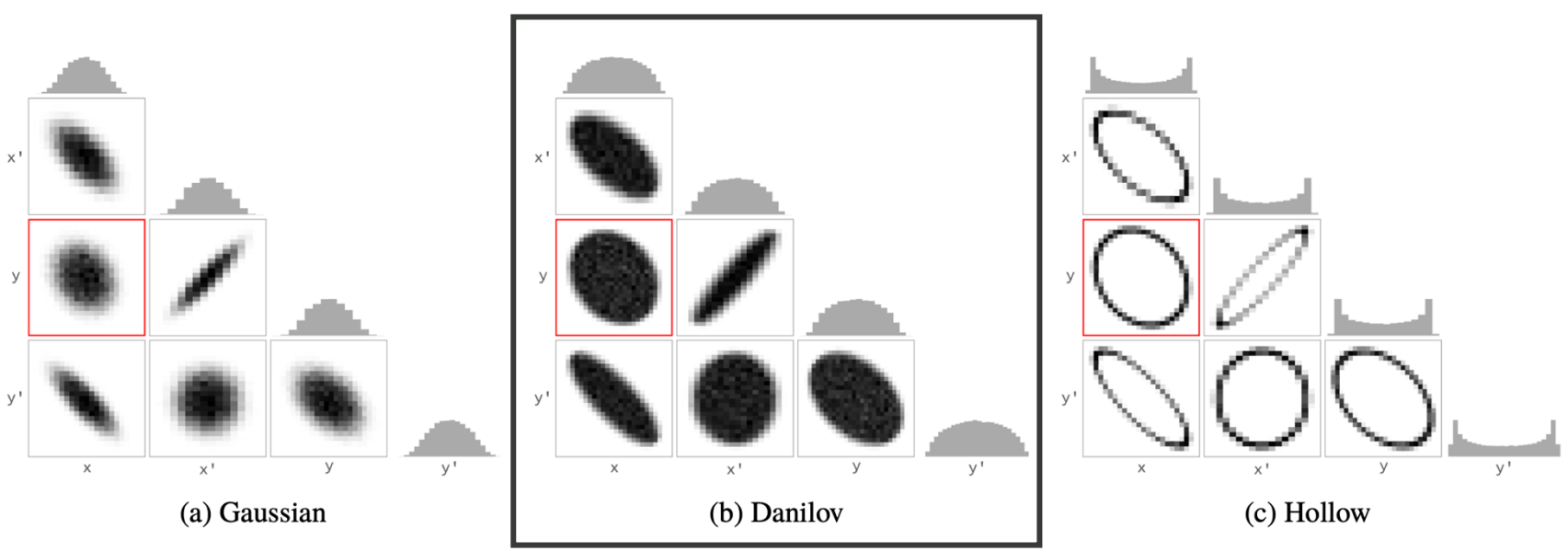
$$\mathbf{x}_{inj}(t) = \sqrt{2J_1(t)}\mathbf{v}_1 e^{-i\psi_1}$$



Normalized



Uniform eigenpainting generates what we will call the Danilov distribution



(a) Gaussian

(b) Danilov

(c) Hollow

This works with space charge — to see why, start from the KV distribution

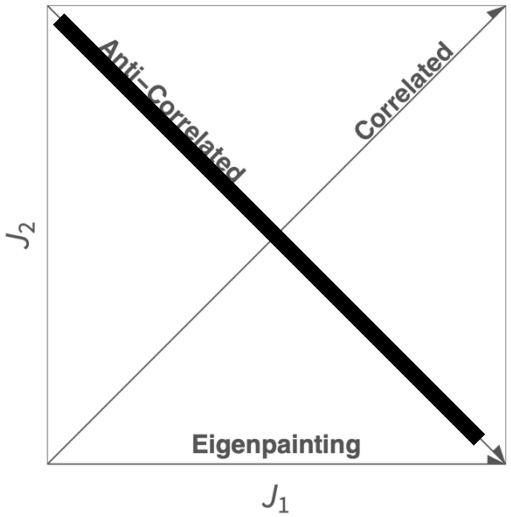
- Generalize KV distribution to include cross-plane coupling:

$$f(\mathbf{x},\mathbf{x}') = \delta\left(1 - \frac{J_1(\mathbf{x},\mathbf{x}')}{\tilde{J}_1} - \frac{J_2(\mathbf{x},\mathbf{x}')}{\tilde{J}_2}\right)$$

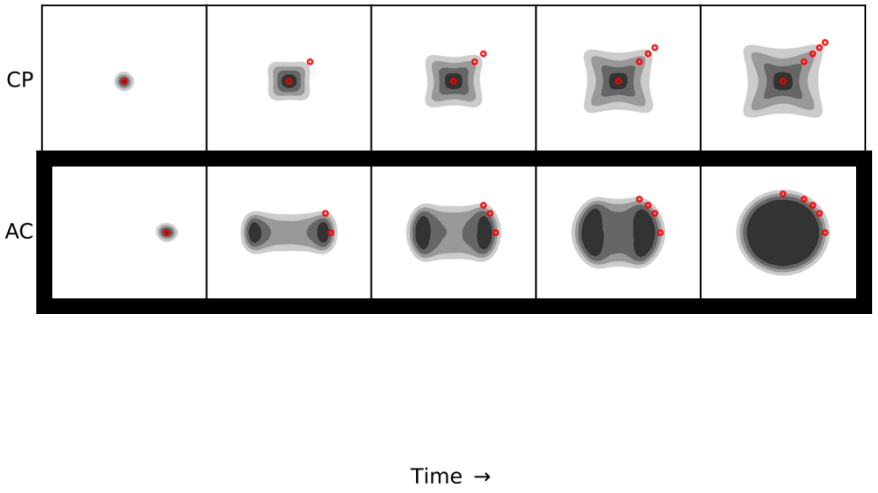
- The functions J_1 and J_2 are single-particle invariants in linear fields. ($\tilde{J}_1$ and $\tilde{J}_2$ are scaling factors.)
- The distribution projects to unform charge density and linear self fields.
- Therefore, J_1 and J_2 are invariants including space charge.
- Therefore, from the Vlasov equation, the distribution is in equilibrium.

We could paint a KV distribution, but only if space charge was negligible

$$f(\mathbf{x},\mathbf{x}') = \delta\left(1 - \frac{J_1(\mathbf{x},\mathbf{x}')}{\tilde{J}_1} - \frac{J_2(\mathbf{x},\mathbf{x}')}{\tilde{J}_2}\right)$$



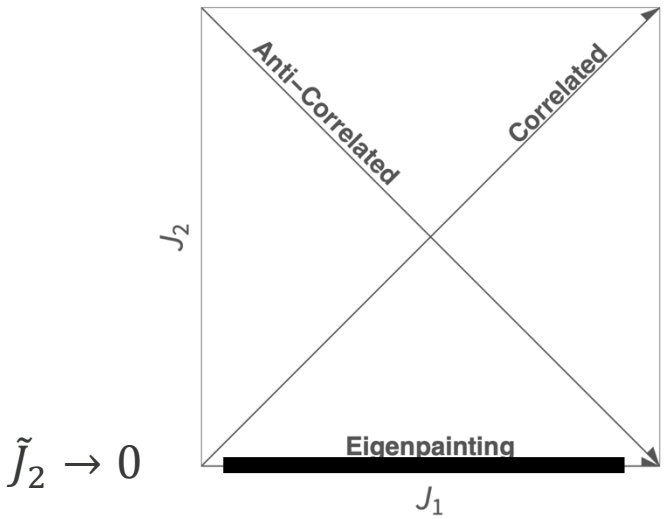
(a) Schematic trajectories for several painting schemes.



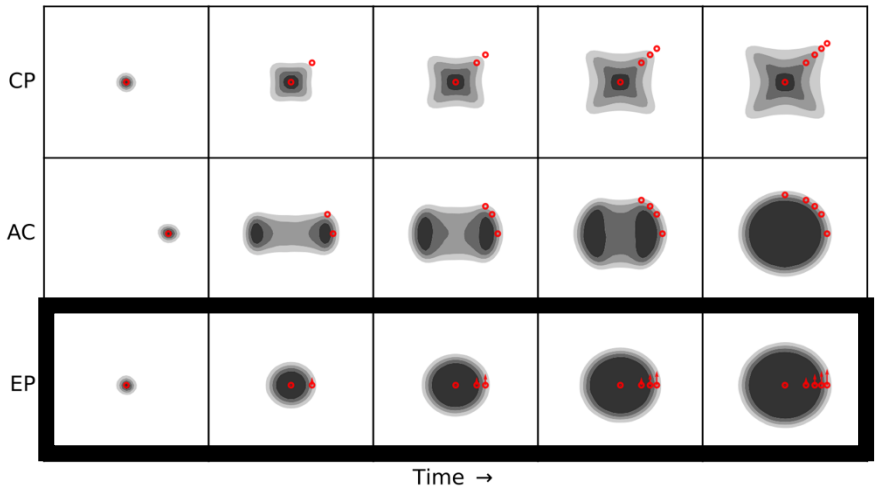
(b) x-y distributions.

But if we paint into only one mode (zero emittance limit)…

$$f(\mathbf{x},\mathbf{x}') \to \theta\left(1-\frac{J_1(\mathbf{x},\mathbf{x}')}{\tilde{J}_1}\right)\delta\big(J_2(\mathbf{x},\mathbf{x}')\big) \qquad (\tilde{J}_2 \to 0)$$

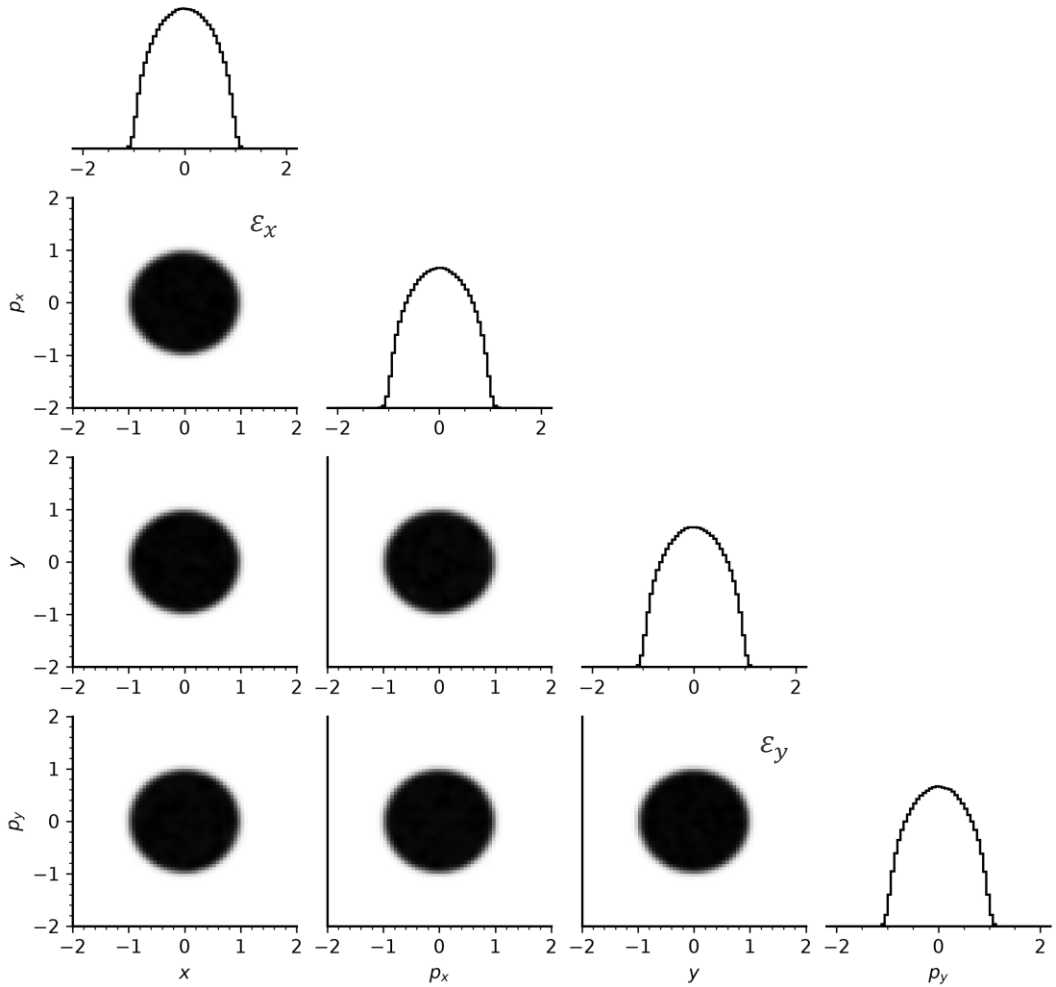


(a) Schematic trajectories for several painting schemes.

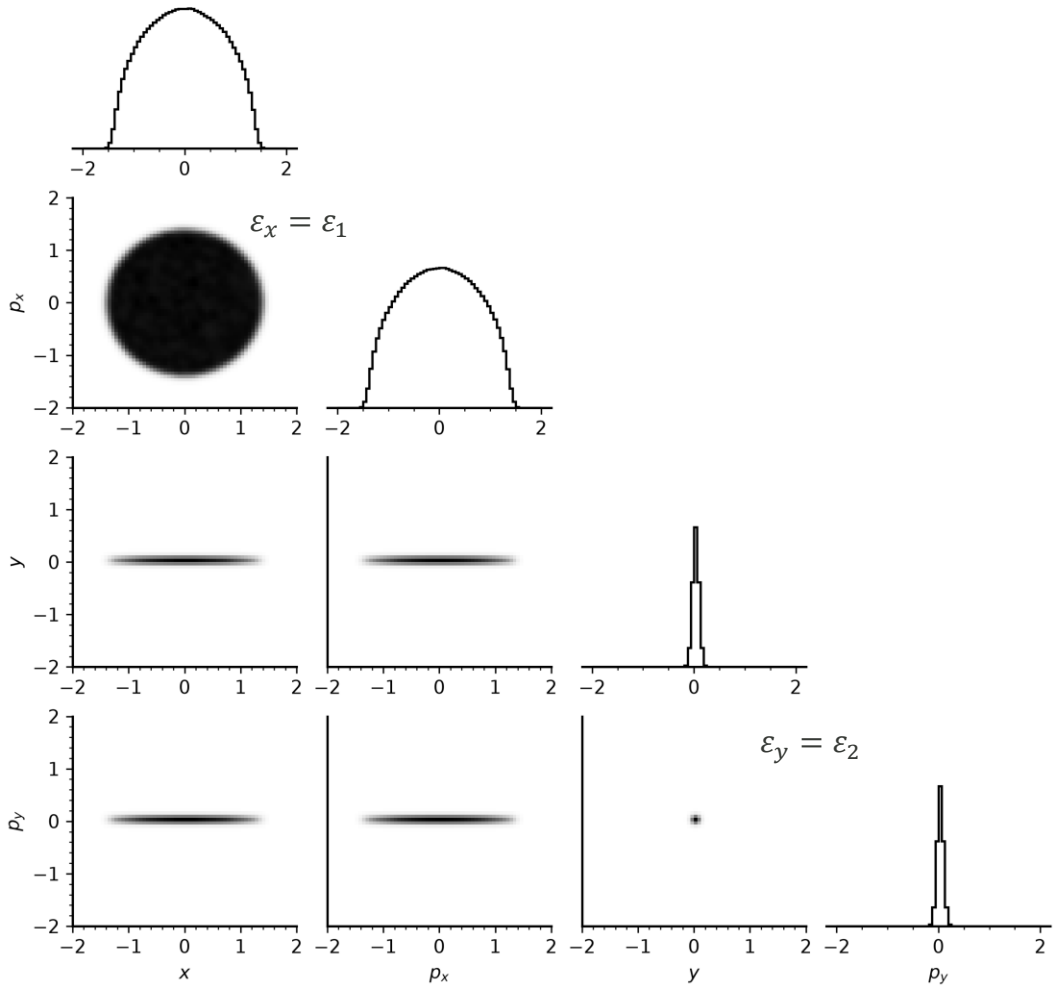


(b) x-y distributions.

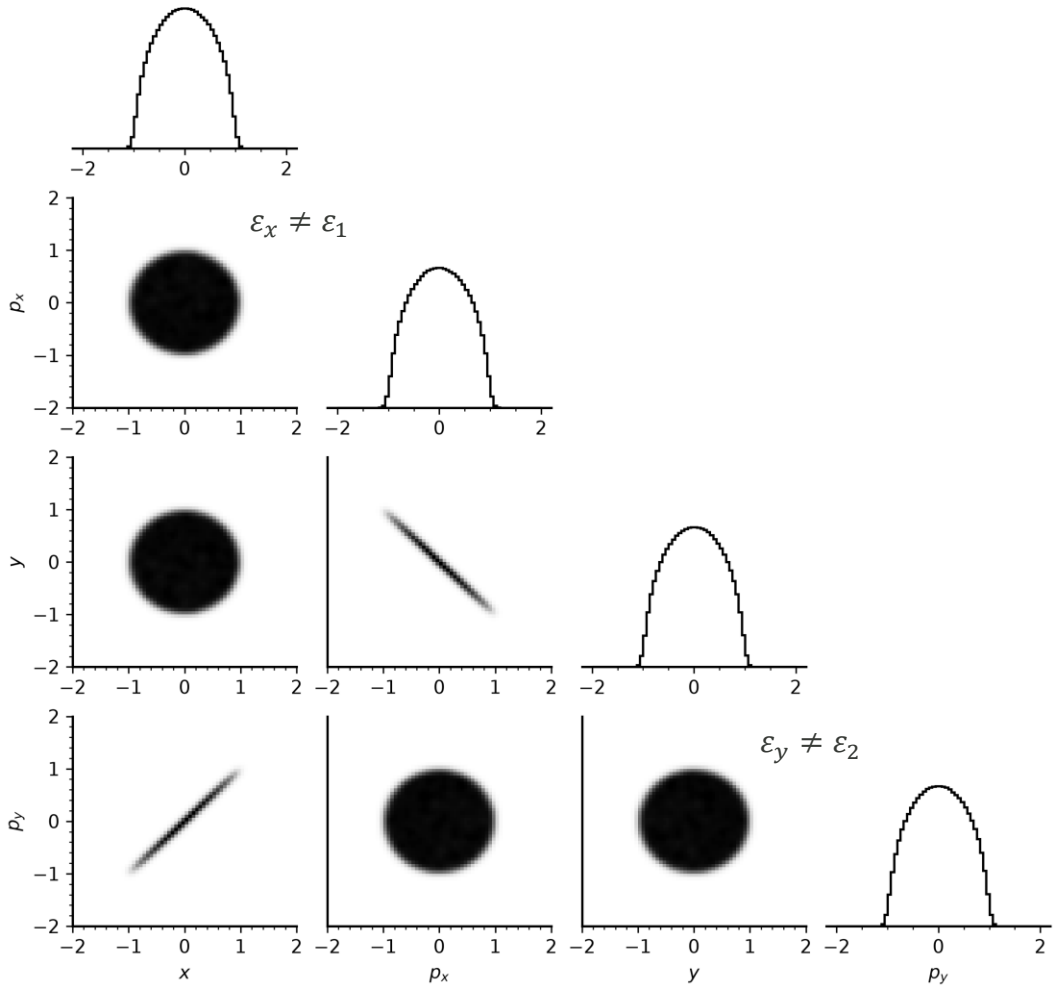
KV distribution



KV distribution + zero-emittance limit = flat Danilov distribution



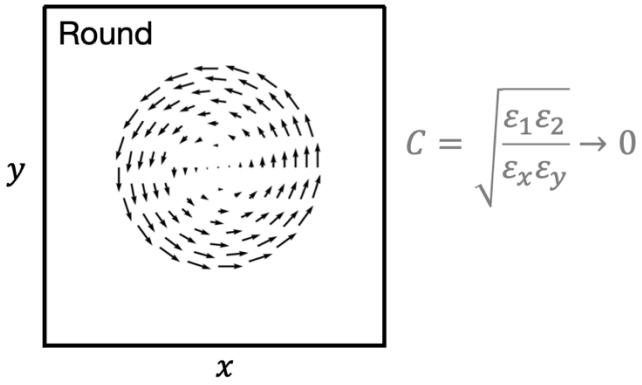
KV distribution + zero-emittance limit + flat-to-round transformation = round Danilov distribution



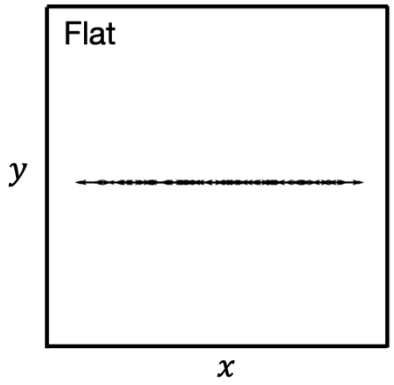
Why would we want to do this?

- Space charge: reduced tune spread, suppressed particle-core effects
- Flat beams for colliders [Phys. Rev. ST Accel. Beams 16, 061002]
- Low-energy hadron cooling [Phys. Rev. E 66, 016503]
- Academic studies: approach “unrealistic” distribution in the lab

Round-to-flat transformation converts large intrinsic emittance ratio to large projected emittance ratio

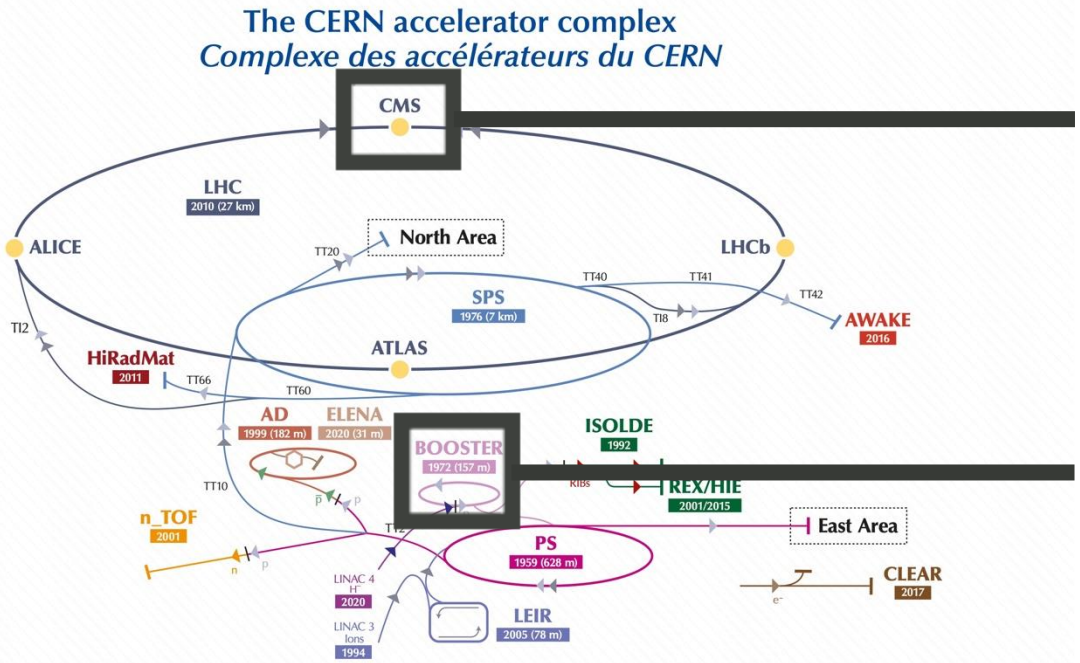


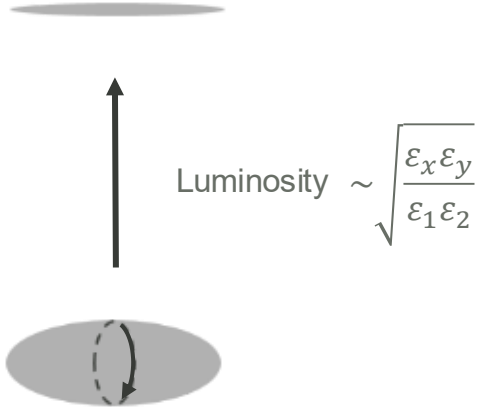
$$C = \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_x \varepsilon_y}} \to 0$$



$$C = \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_x \varepsilon_y}} \to 1$$

Idea: flatten beam after acceleration to boost collider luminosity

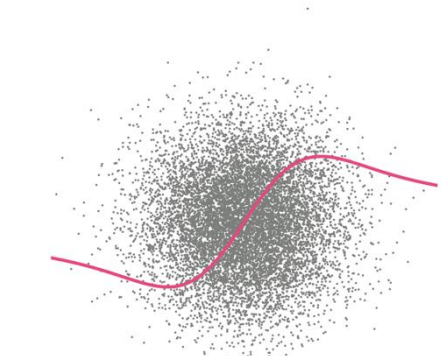


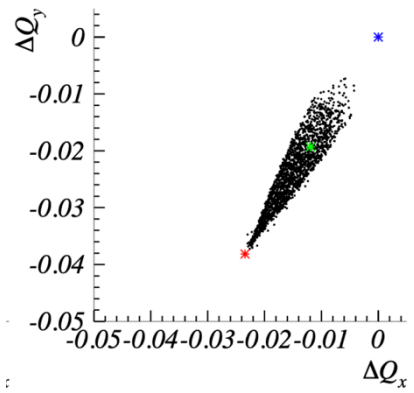


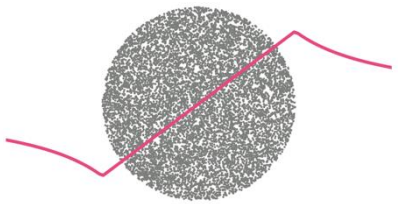
Circular modes for flat beams in the LHC

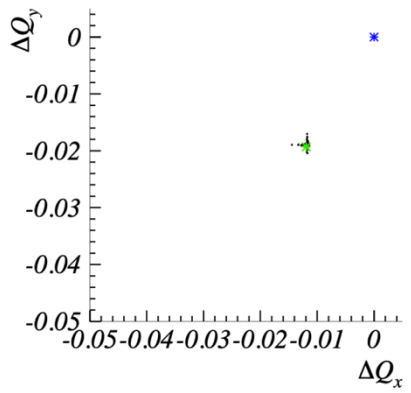
A. Burov
Phys. Rev. ST Accel. Beams **16**, 061002 – Published 24 June 2013

Uniform spatial distribution minimizes tune shift and eliminates tune spread

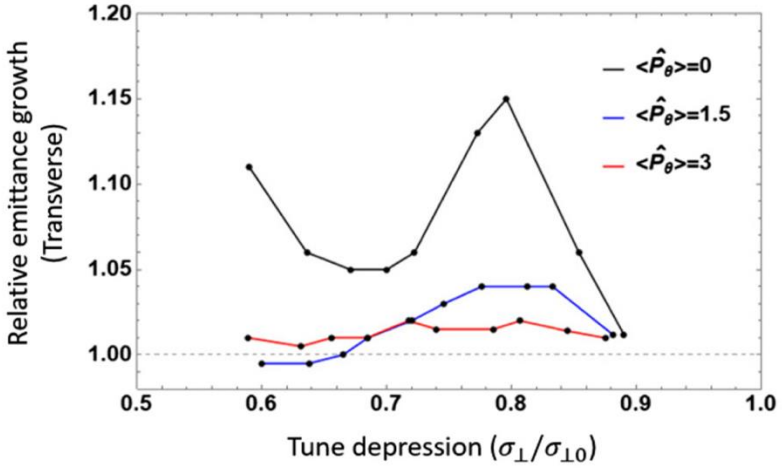


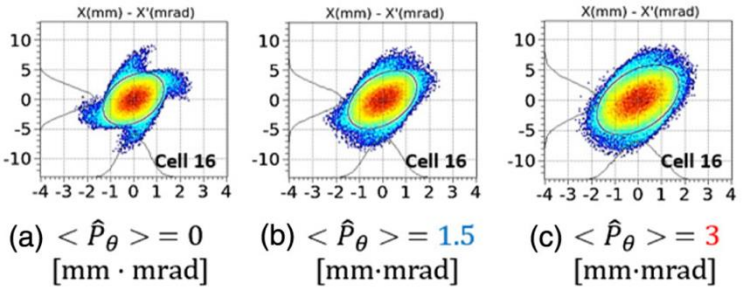






Angular momentum may help mitigate halo formation from particle-core resonance





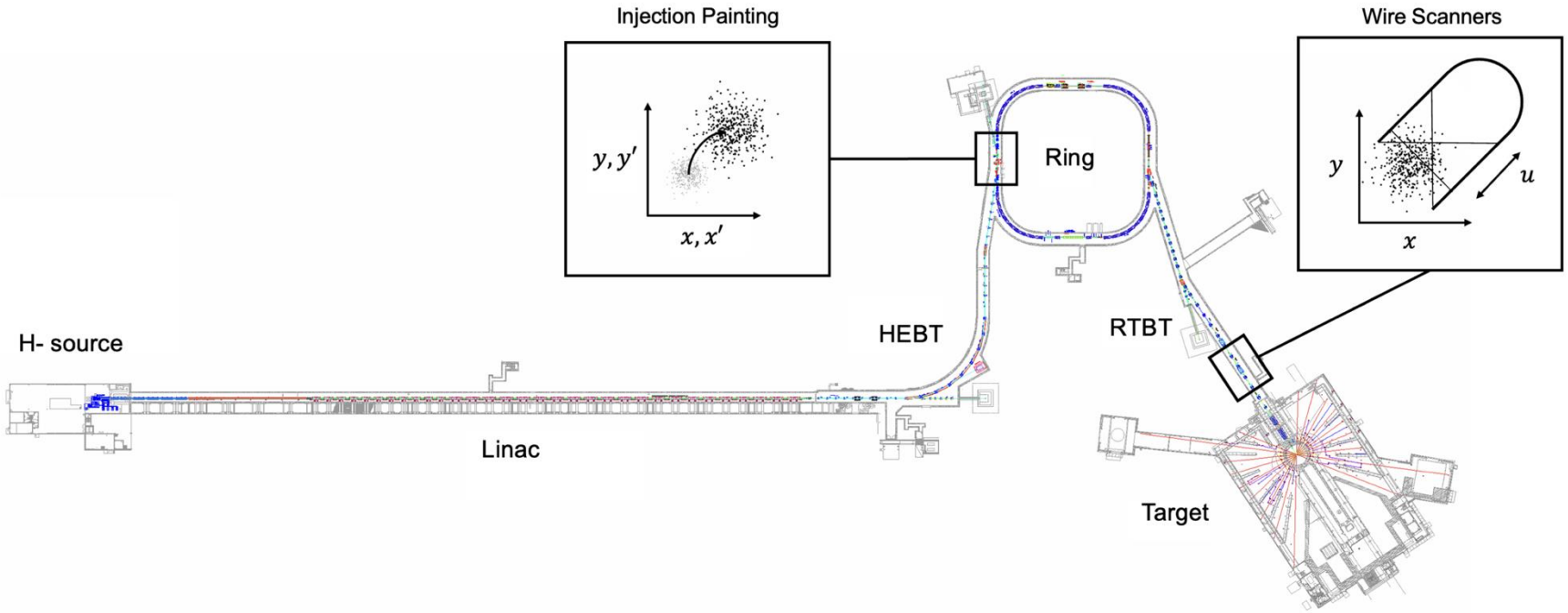
$$r'' + k(s)r - P_\theta^2/r^3 = f(r)$$

Effects of beam spinning on the fourth-order particle resonance of 3D bunched beams in high-intensity linear accelerators

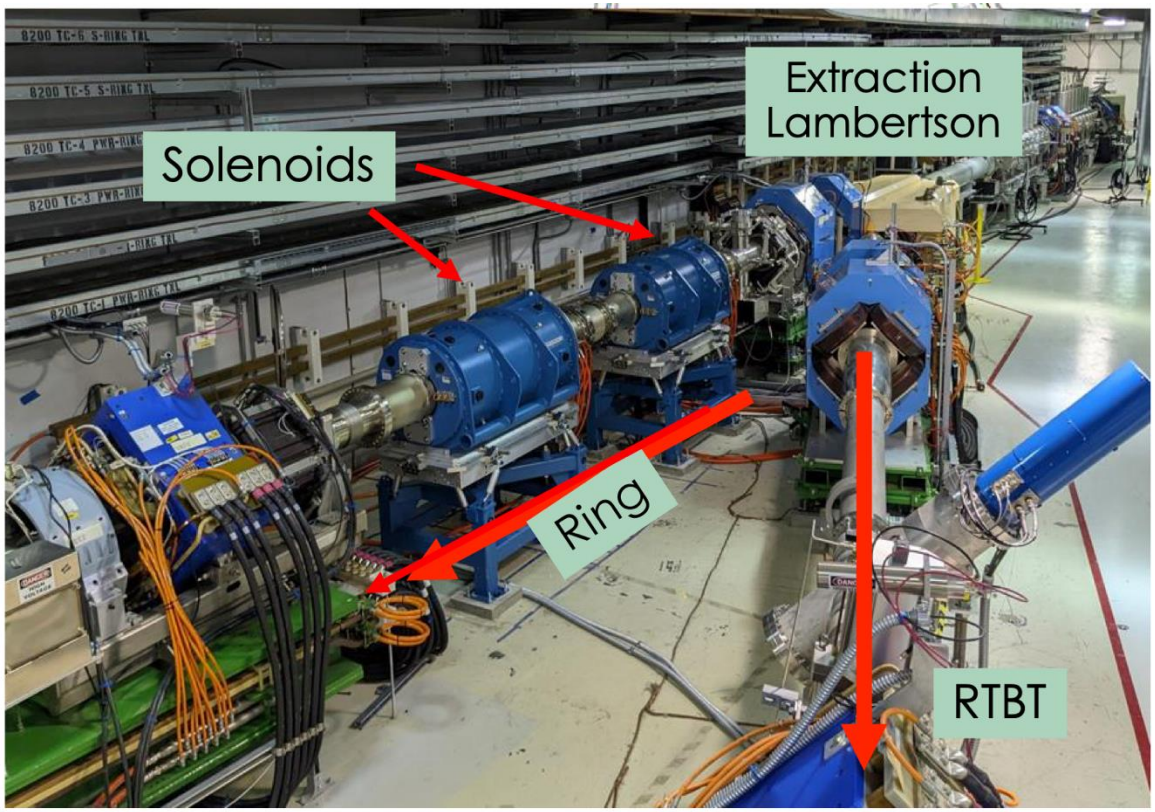
Yoo-Lim Cheon, Seok-Ho Moon, Moses Chung, and Dong-O Jeon
Phys. Rev. Accel. Beams **25**, 064002 – Published 10 June 2022

SNS experiments

Initial experiments have attempted to implement eigenpainting in the SNS AR

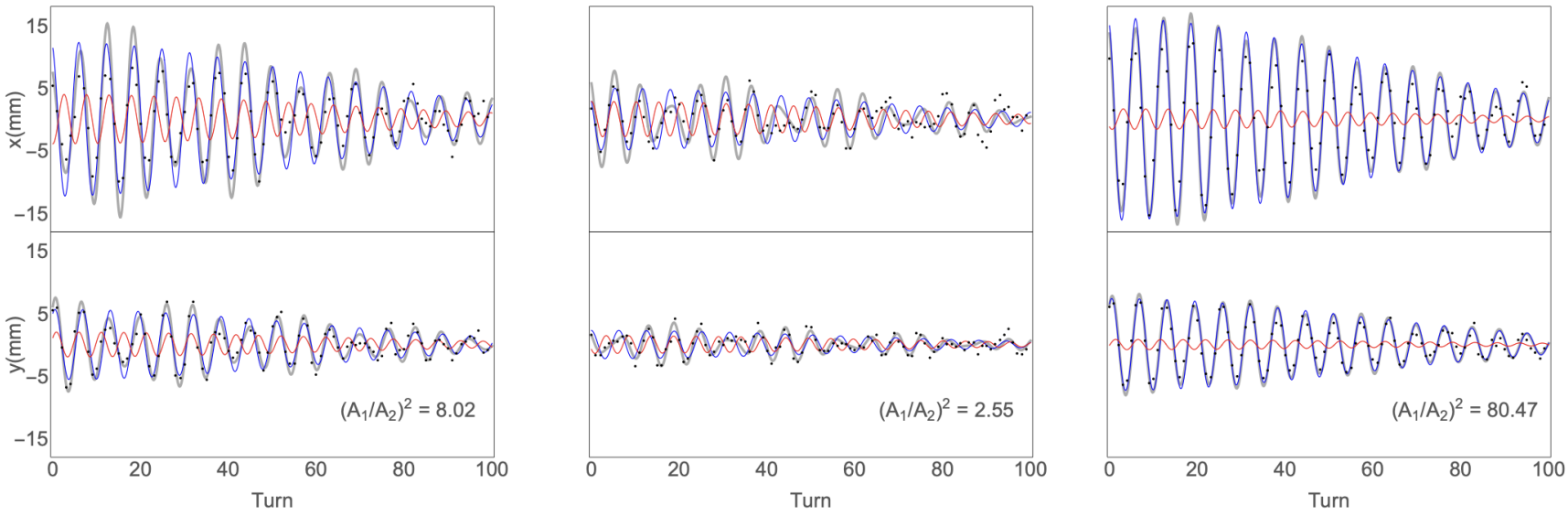


Two solenoids generate linear coupling/non-planar modes in the ring



Kicker amplitudes were hand-tuned to inject along eigenvector

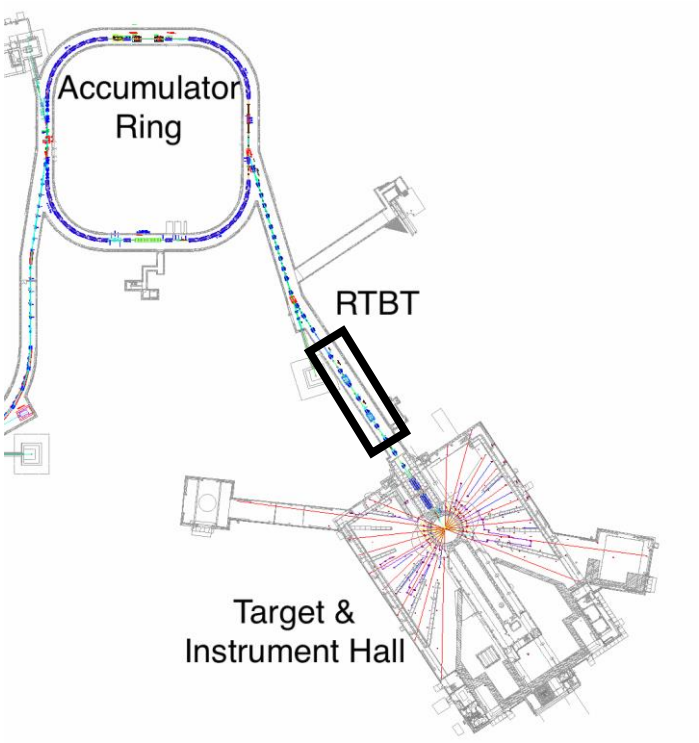
$$\mathbf{x}^j(n) = \sum_{k=1,2} A_k e^{-\gamma_k n^2 - i(2\pi n \nu_k + \Psi_k)} \mathbf{v}_k^j(n) + \mathbf{c}^j,$$

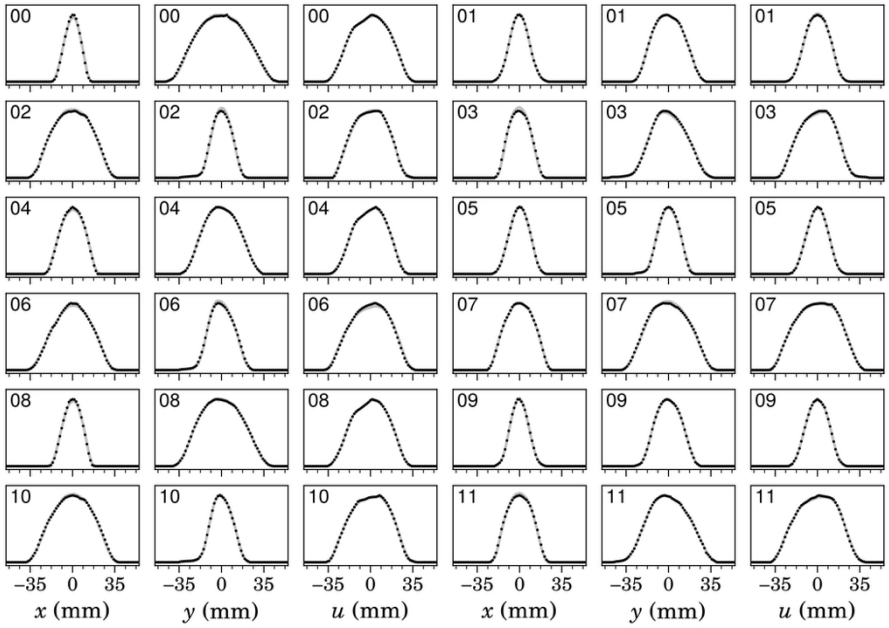


(a) Calibration Data (b) Injection at $t = 0$ (c) Injection at $t = T_f$

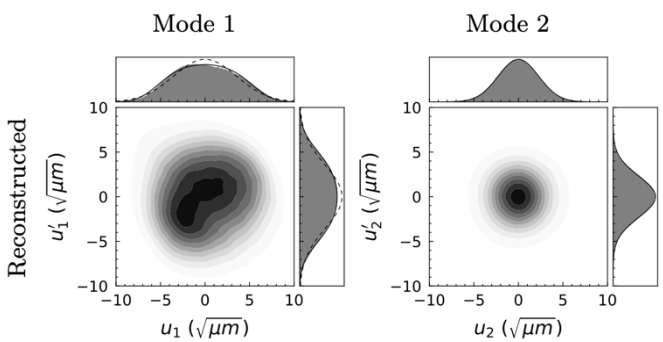
Connect with sqrt(t) waveform

Wire scanners measure 1D profiles of painted distribution

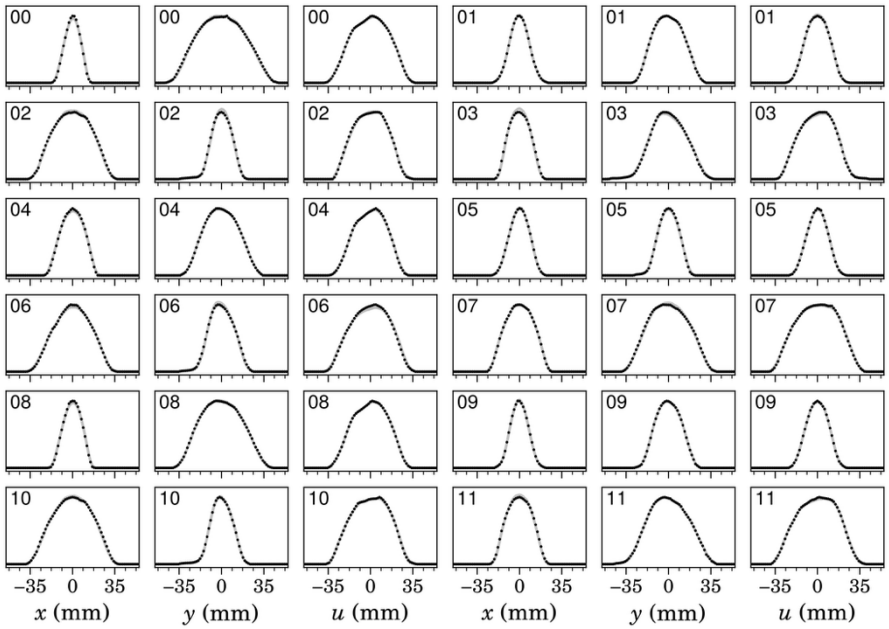




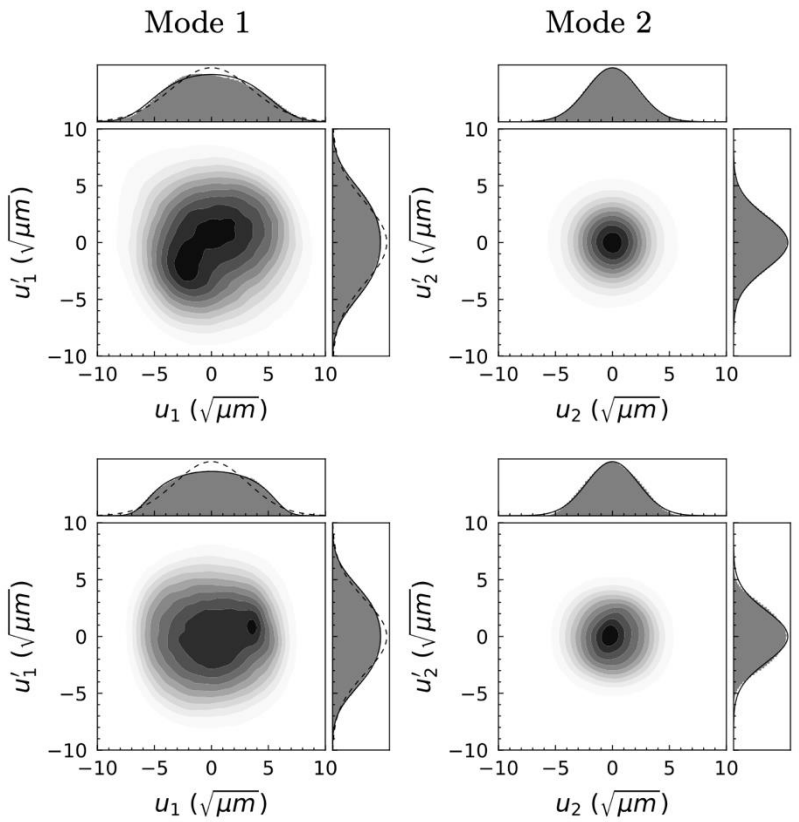
MENT generates best-guess 4D phase space distribution from data + accelerator model



$$\max_f S[f] \text{ s.t. } G_k[f] = 0$$



Simulations produce correct emittances and qualitative agreement with reconstruction

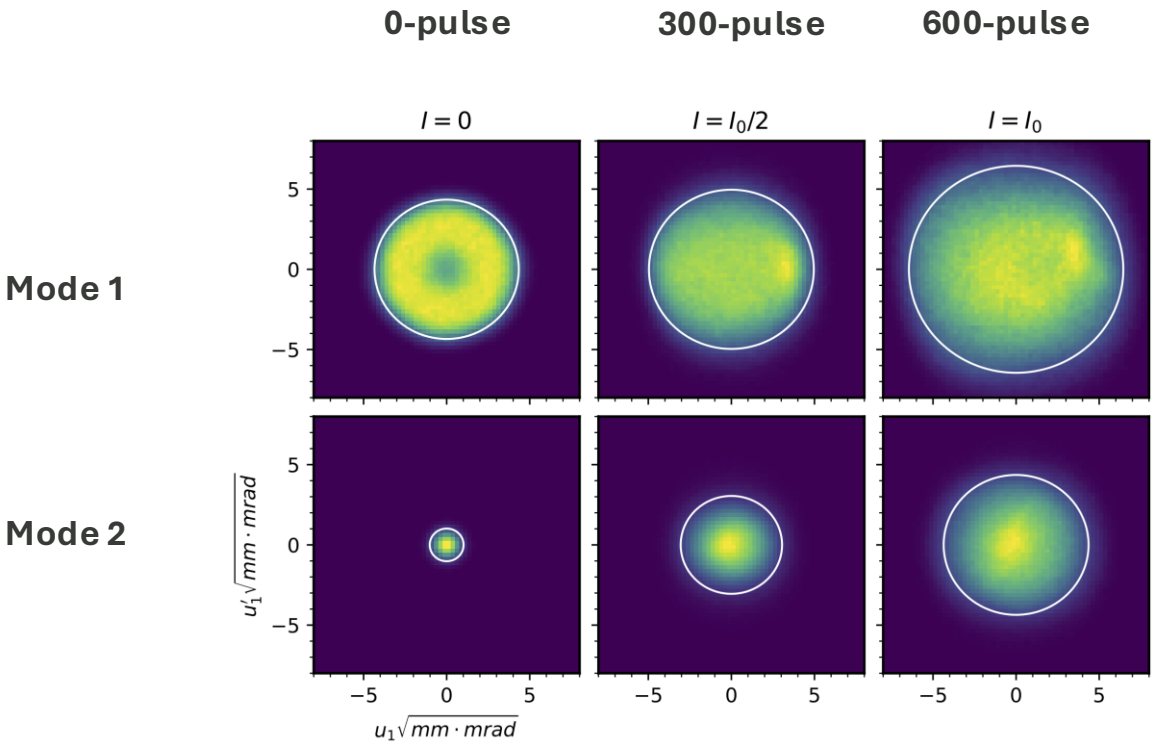


$\varepsilon_1/\varepsilon_2 \approx 2.4$

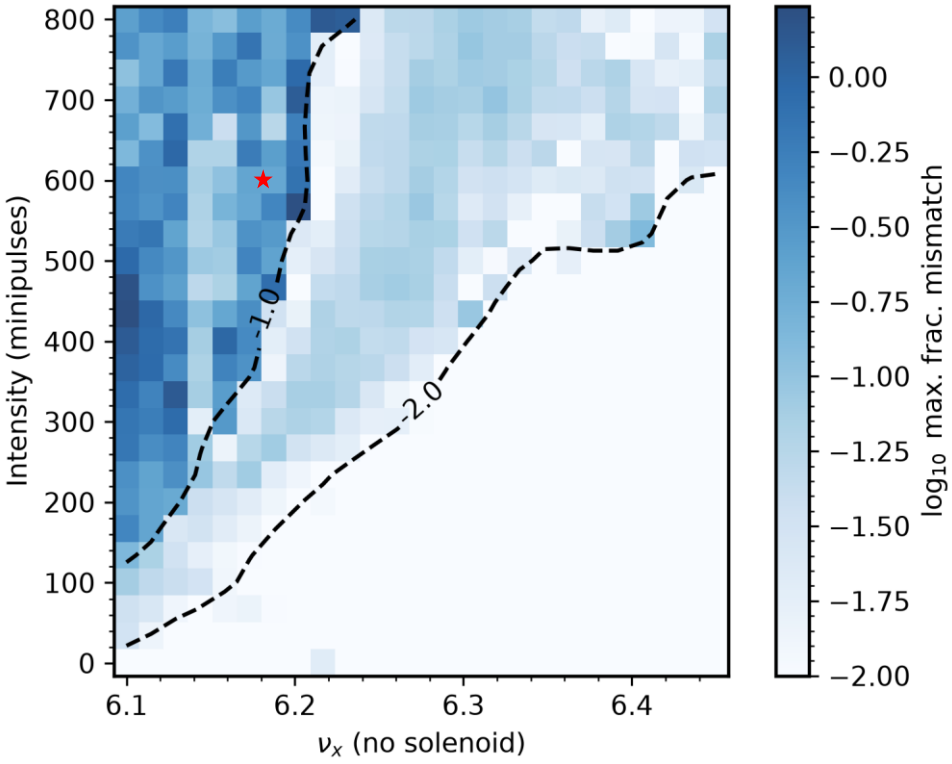
$\varepsilon_1/\varepsilon_2 \approx 2.3$

Analysis and current work

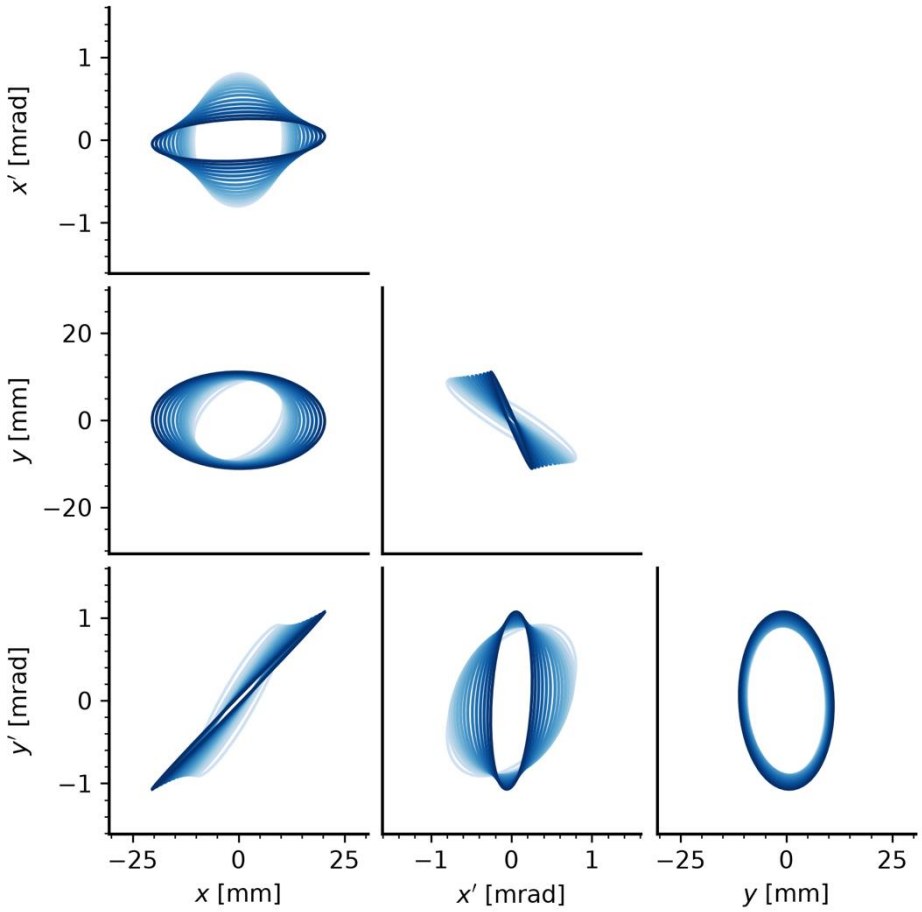
Simulations suggest 4D emittance growth was driven by nonlinear space charge forces



First issue: can’t find periodic envelope at beam intensity + tunes used in experiment

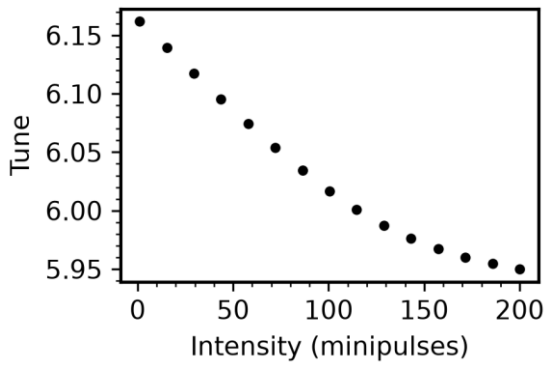


Second issue: space charge changes eigenvectors

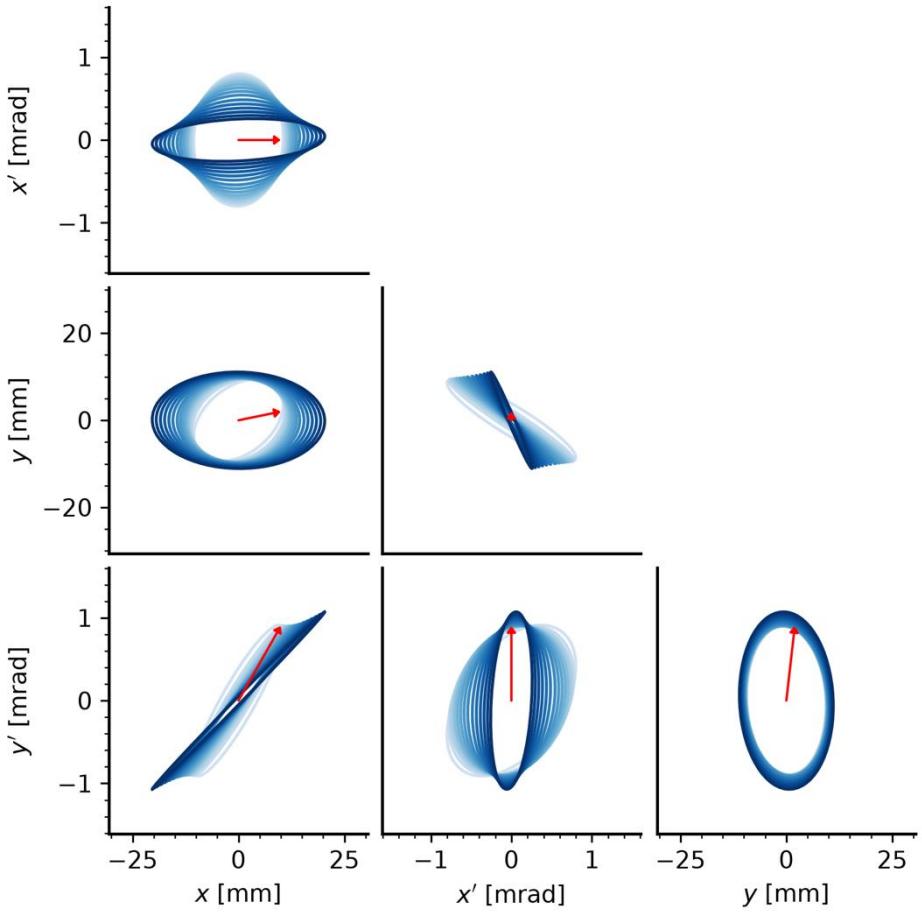


Modified eigenvectors calculated from periodic beam envelope.

$$\boldsymbol{\Sigma} = \mathbf{M}\boldsymbol{\Sigma}\mathbf{M}^T$$
$$\boldsymbol{\Sigma}\mathbf{U}\hat{\mathbf{v}} = i\epsilon\hat{\mathbf{v}}$$

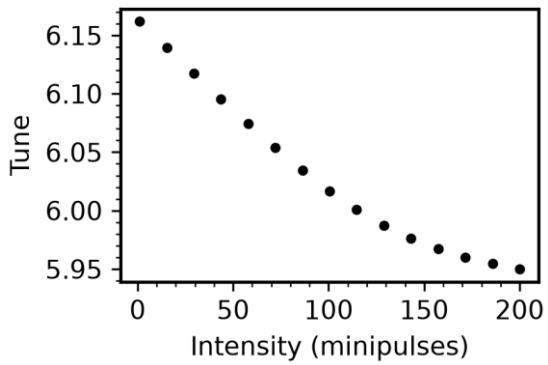


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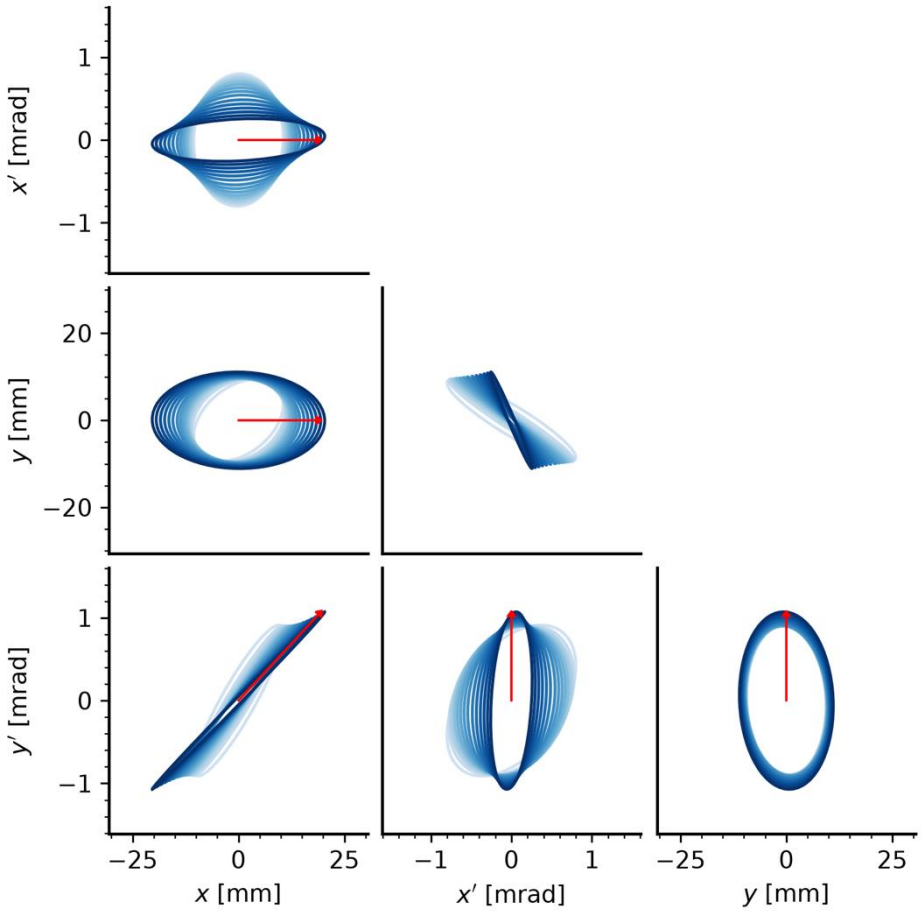


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$$\boldsymbol{\Sigma} = \mathbf{M}\boldsymbol{\Sigma}\mathbf{M}^T$$
$$\boldsymbol{\Sigma}\mathbf{U}\hat{\mathbf{v}} = i\epsilon\hat{\mathbf{v}}$$

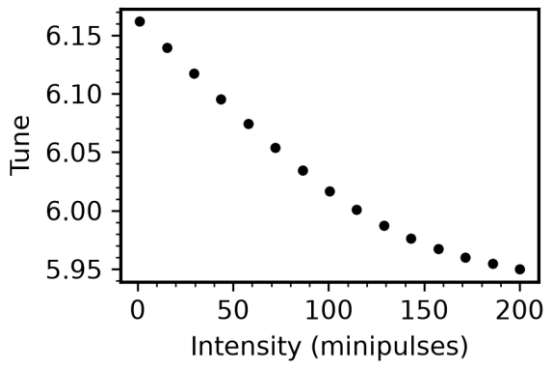


Second issue: space charge changes eigenvectors



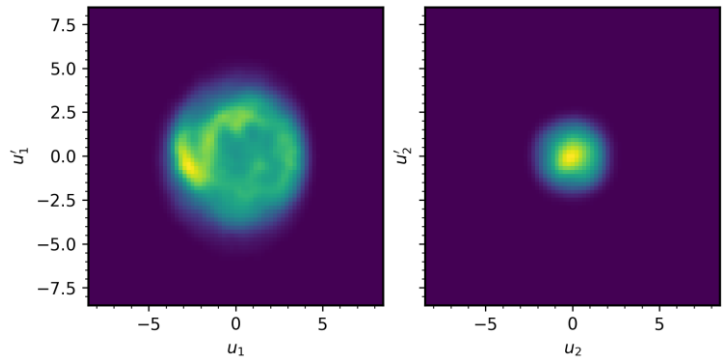
Modified eigenvectors calculated from periodic beam envelope.

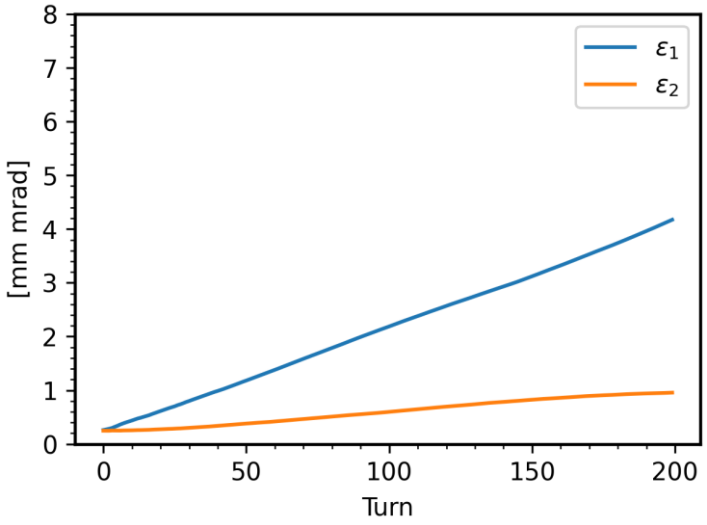
$$\boldsymbol{\Sigma} = \mathbf{M}\boldsymbol{\Sigma}\mathbf{M}^T$$
$$\boldsymbol{\Sigma}\mathbf{U}\hat{\mathbf{v}} = i\epsilon\hat{\mathbf{v}}$$



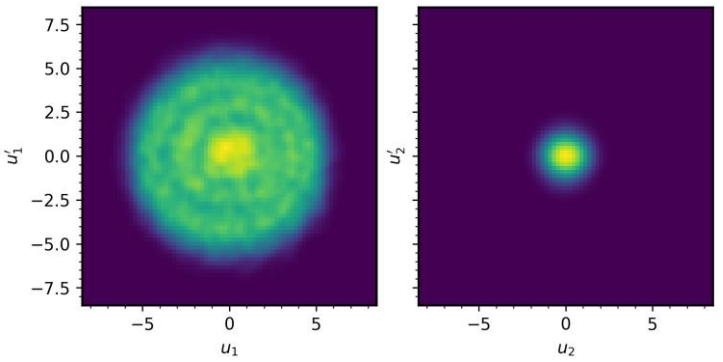
SC-modified eigenvectors could generate much larger emittance ratio (>10) within kicker limits

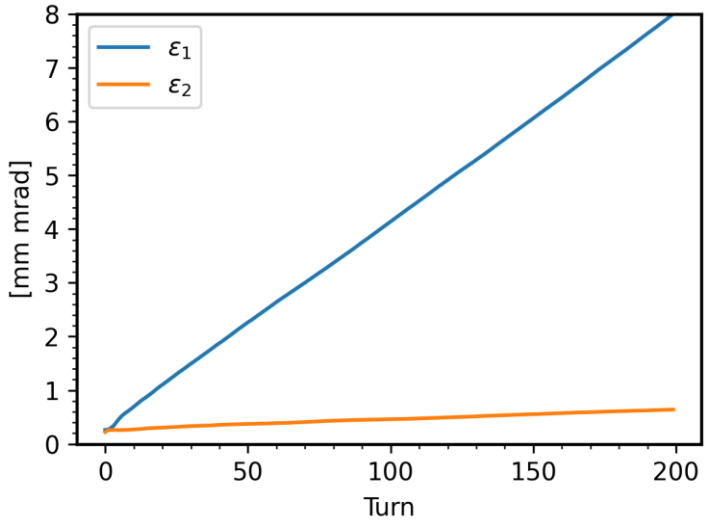
$\mathbf{x} \propto \mathbf{v}$



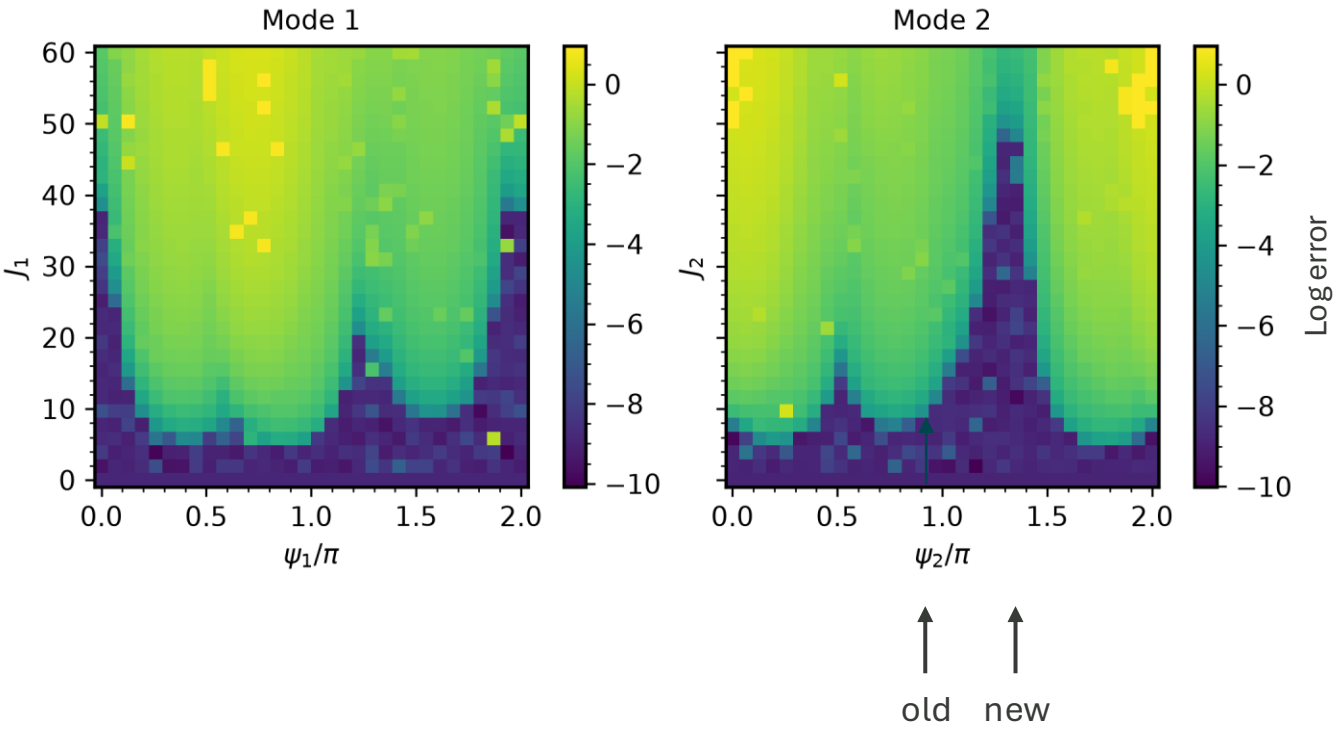


$\mathbf{x} \propto \hat{\mathbf{v}}$



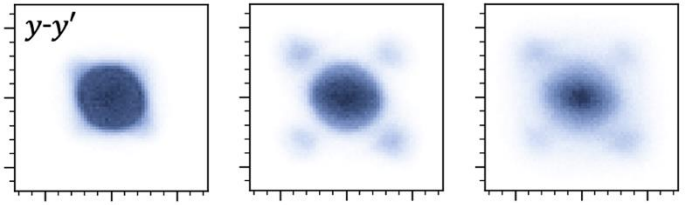


Also identified phase which could enable larger amplitude within kicker reach (but requires x' > 0)

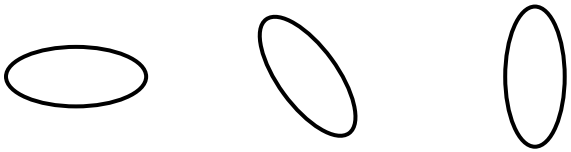


Currently exploring particle-core dynamics and coherent instabilities for Danilov distribution

Structure resonance



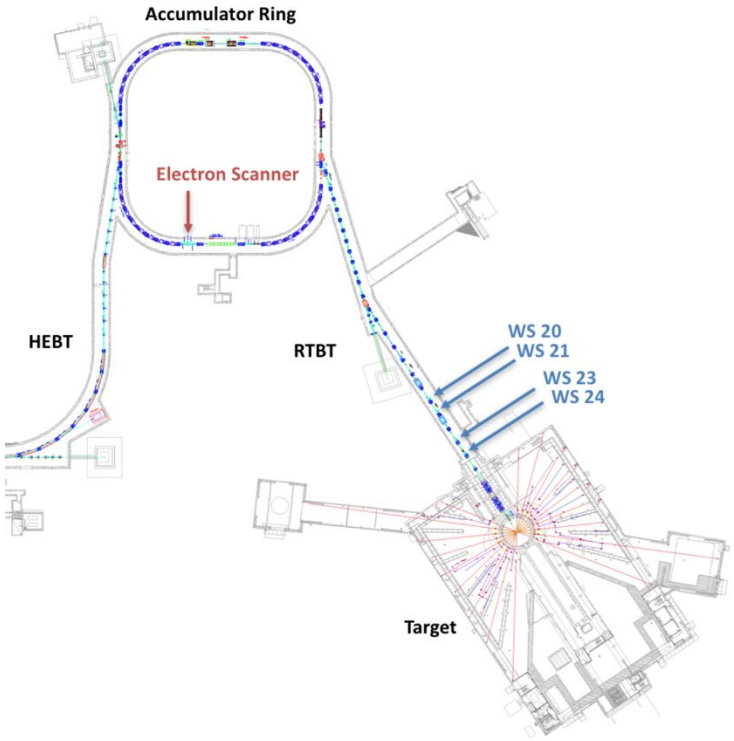
Coherent instability (second order)

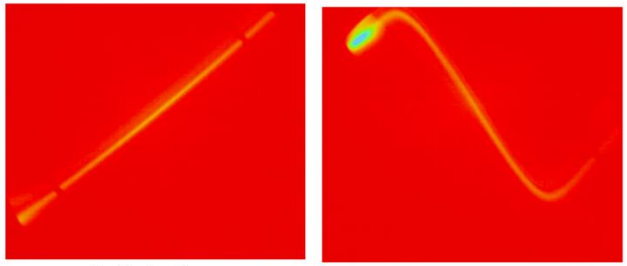


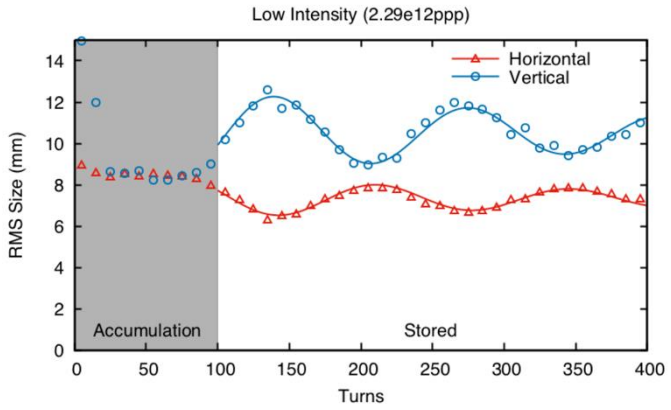
Coherent instability (higher order)



Online optimization of kicker waveforms from electron scanner data





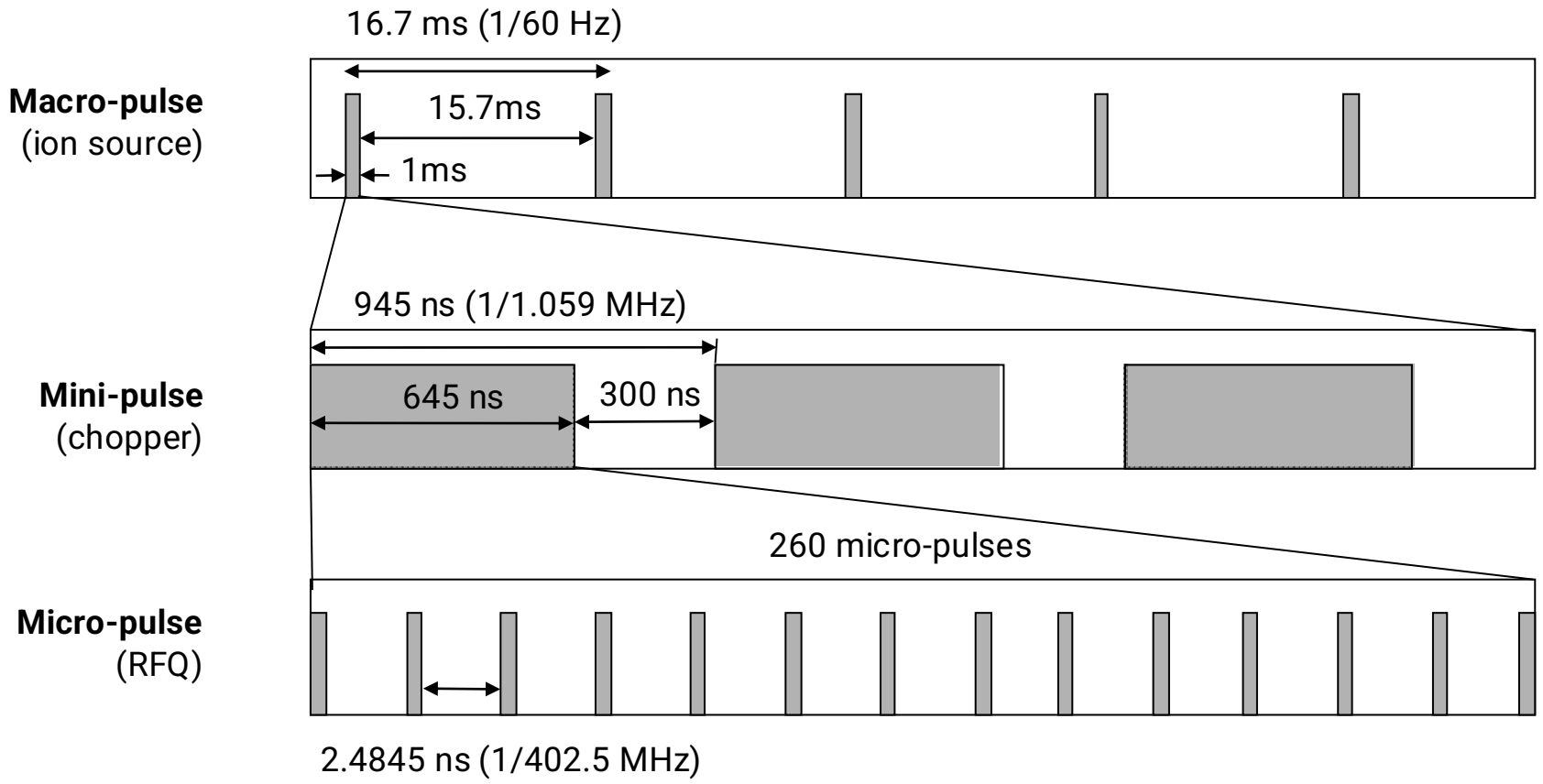


Conclusion

- Paper: Phys. Rev. Lett. **136**, 075002
- Experimental goals:
  - Automate closed-orbit measurement and kicker optimization.
  - Directly optimize kicker waveforms using electron scanner profiles.
  - Demonstrate significant (10?) emittance ratio using modified painting path.
  - Study beam dynamics before/after painting.
- Research topics/questions:
  - Space-charge-driven halo, losses, and coherent instabilities (“Gluckstern modes”)
  - 4D emittance preservation during long-term storage and acceleration
  - Do smooth equilibria exist and, if so, can they be painted in a ring?

Extra slides

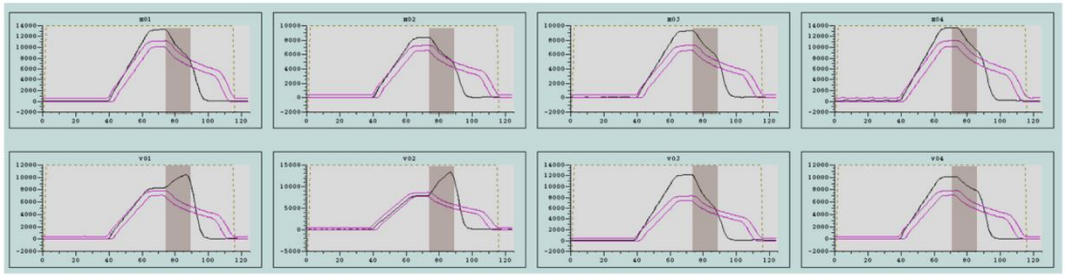
SNS pulse structure



Kicker waveforms and closed-orbit trajectory

Horizontal Kickers

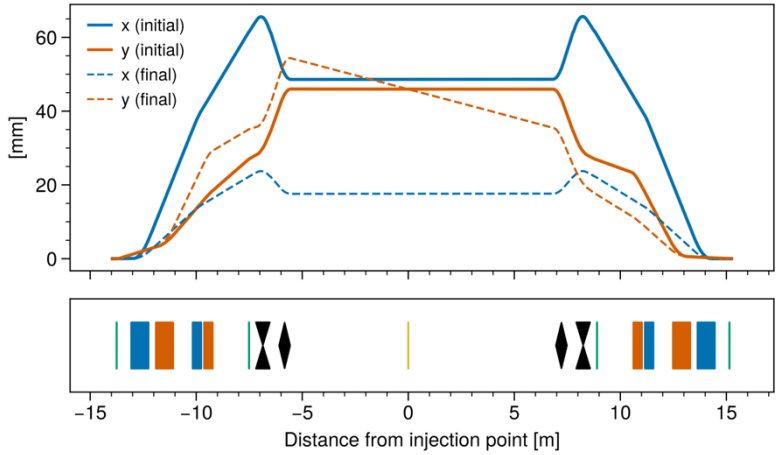
Vertical Kickers



Experiment WF

Production WF

Example trajectory in kicker region



Anti-correlated painting with and without space charge forces

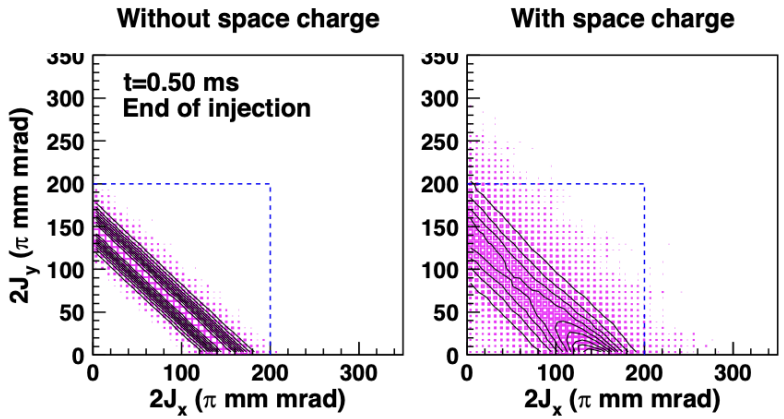
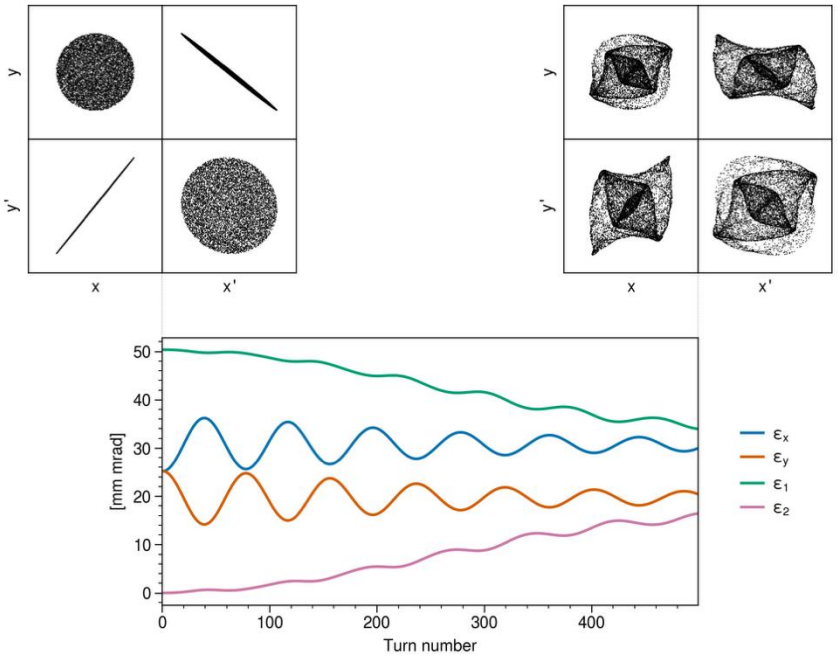


FIG. 8. Numerical simulation results; scatter plots of the single-particle actions (J_y *vs* J_x) from the beginning to the end of injection, calculated for anticorrelated painting with $\epsilon_{tp} = 200\pi$ mm mrad without (left) and with (right) space charge.

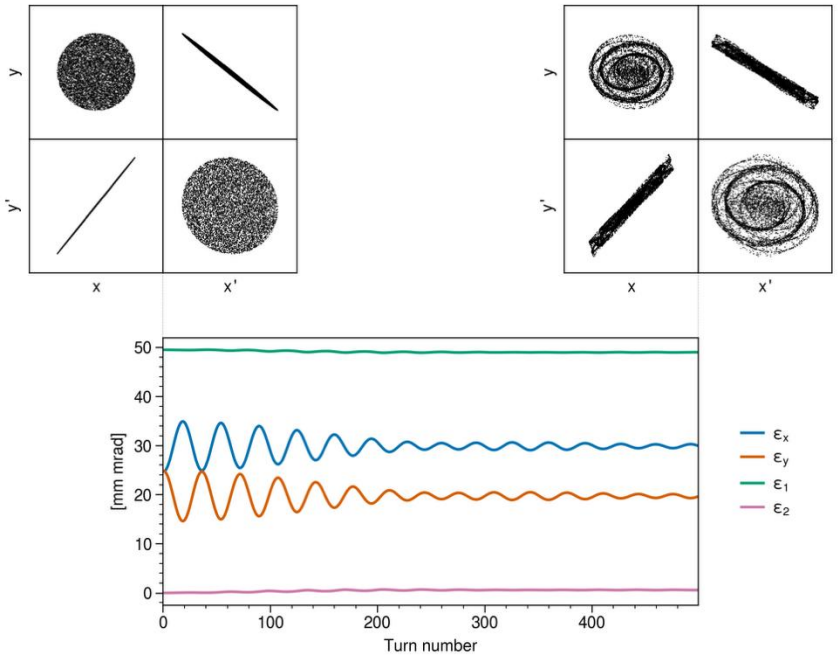
H. Hotchi. *Effects of the Montague resonance on the formation of the beam distribution during multiturn injection painting in a high-intensity proton ring*. PRAB **23**, 050401 (2020)

Solenoids used to reduce emittance exchange near difference resonance

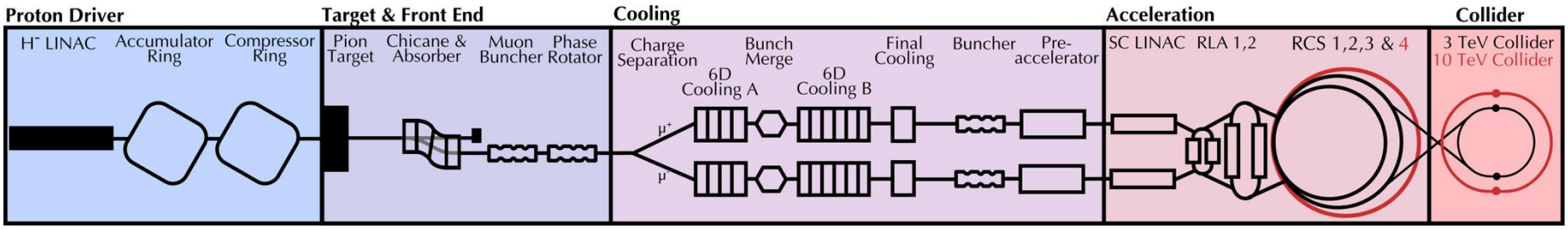
Solenoid OFF



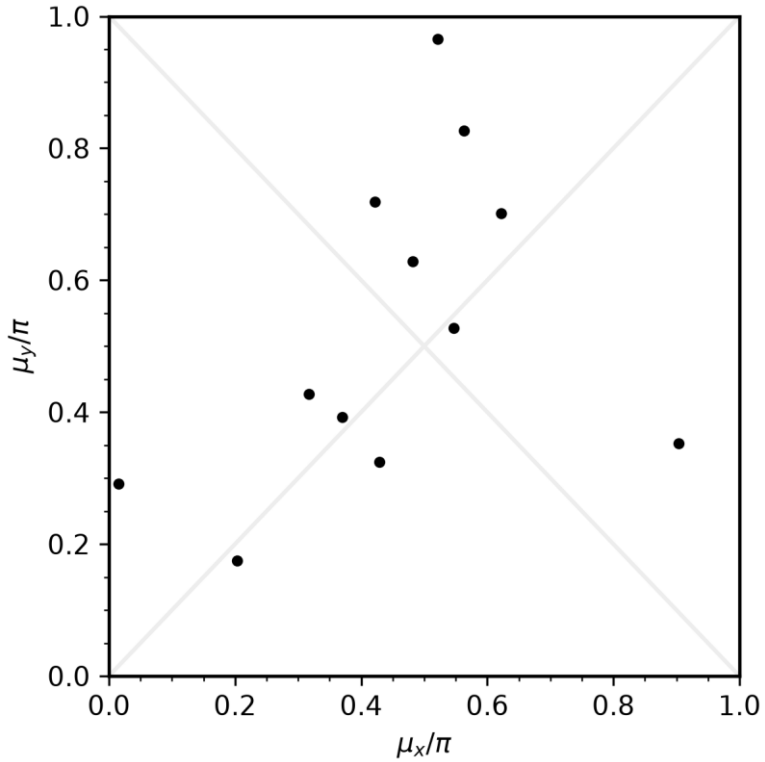
Solenoid ON



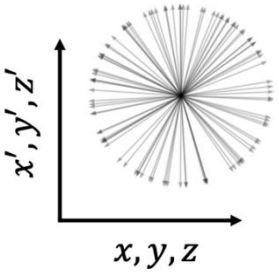
Could eigenpainting reduce beam loss during bunch compression?



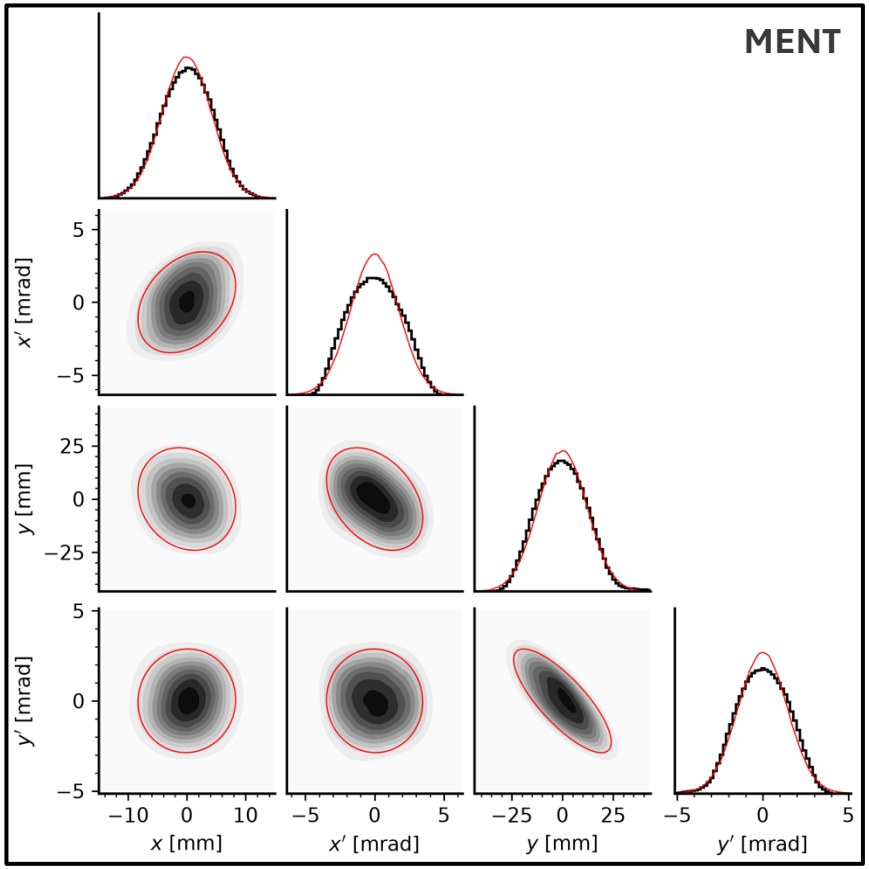
Spread of phase advances in 4D measurement

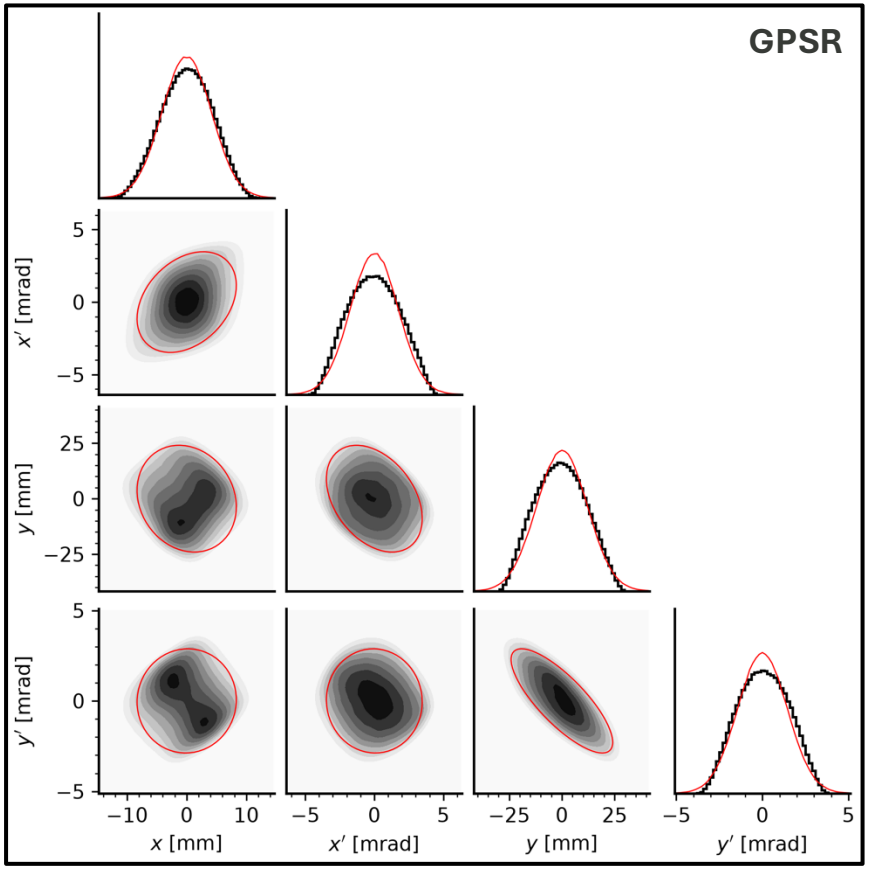


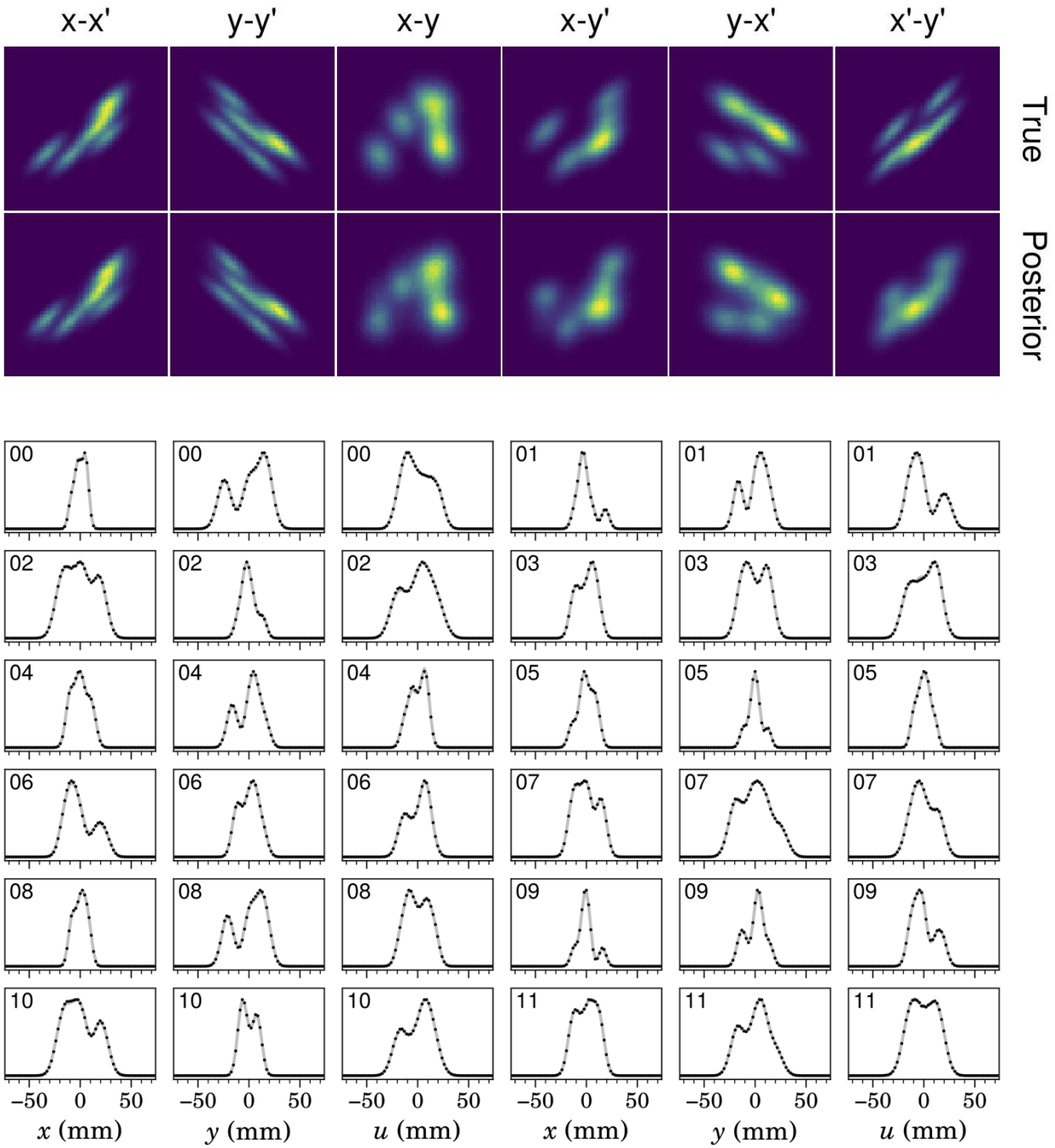
- Result for 2D projections from Hock and Wolski: evenly distribute points in μ_x-μ_y.
- Better/more general: dot products?



Careful!


MENT
x′ [mrad]
y [mm]
y′ [mrad]
x [mm]
x′ [mrad]
y [mm]
y′ [mrad]


GPSR
x′ [mrad]
y [mm]
y′ [mrad]
x [mm]
x′ [mrad]
y [mm]
y′ [mrad]


x-x'
y-y'
x-y
x-y'
y-x'
x'-y'
True
Posterior
00
01
02
03
04
05
06
07
08
09
10
11
−50 0 50
x (mm)
y (mm)
u (mm)