

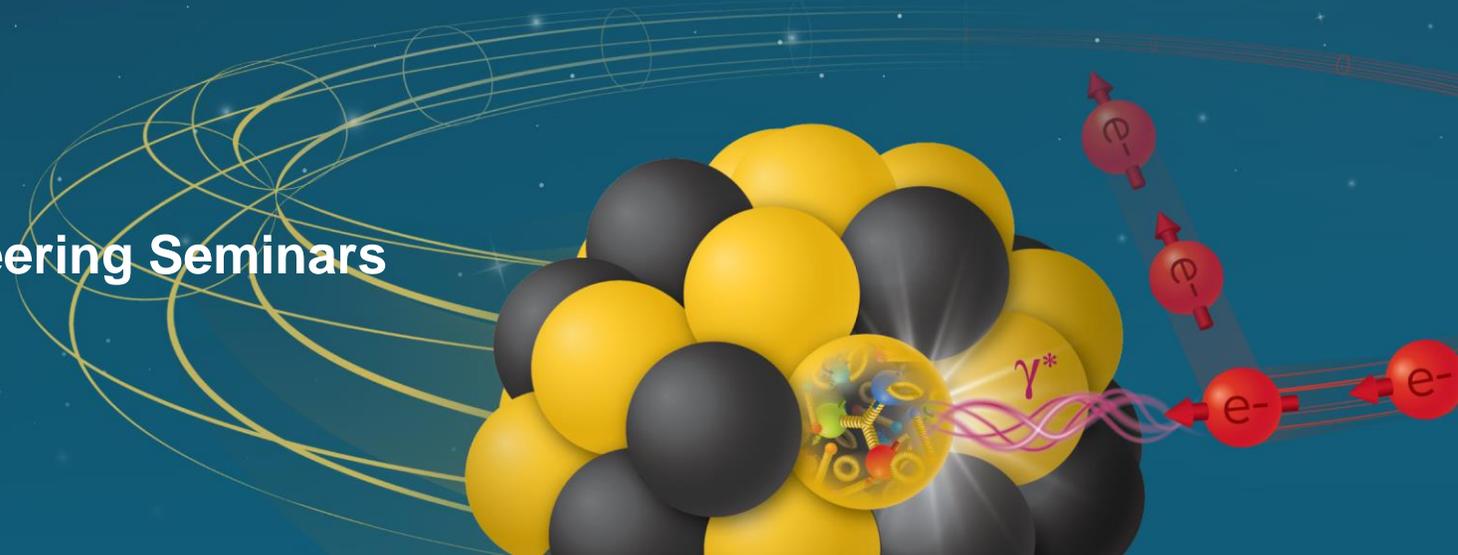
Beam-Beam Effects In Future EIC

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FRIB Accelerator Physics and Engineering Seminars
Feb 09, 2024

Electron-Ion Collider

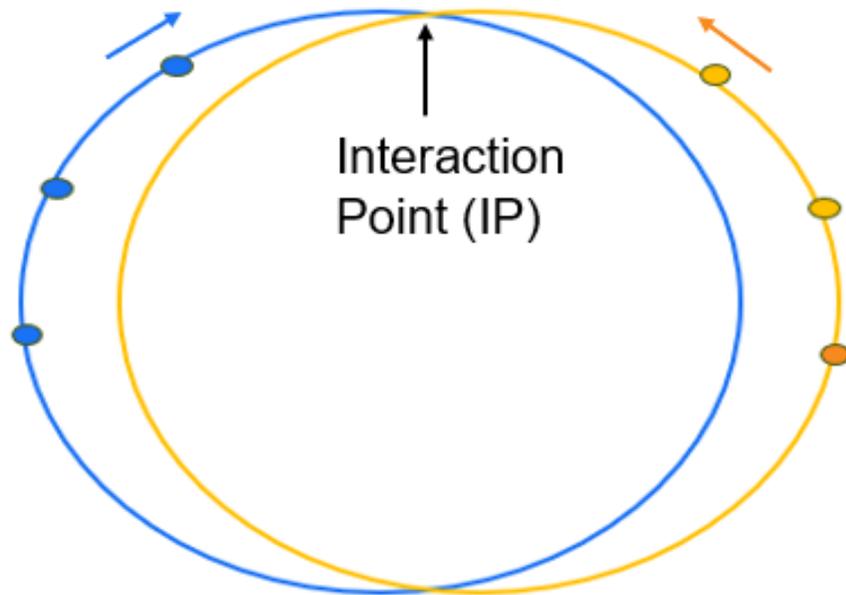


Outline

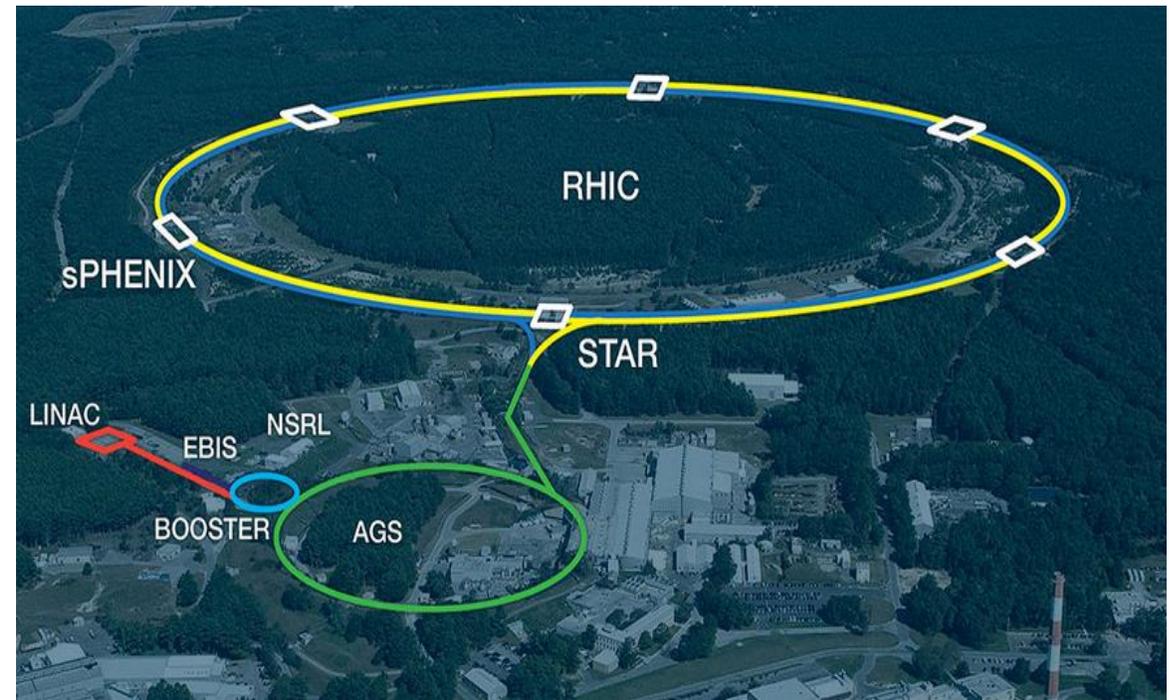
- Introduction to Beam-Beam Interaction
- Introduction to Electron-Ion Collider
- Beam-Beam Effects in EIC
- Summary

Colliders

- **Beam-beam interaction and its effects are only related to colliders.**
- **Particle accelerator:** a machine to accelerate charged particles to high energy.
- **Collider:** a particle accelerator to collide opposing bunches of charged particles.



Schematic plot of a circular collider



Relativistic heavy Ion Collider (RHIC)

Luminosity

- Event rate of a reaction:

$$R = L\sigma$$

- **Luminosity** definition:

$$L = \frac{N_1 N_2 f_c}{4\pi\sigma_x\sigma_y} H(\beta_{x,y}, \sigma_l, \theta, \dots)$$

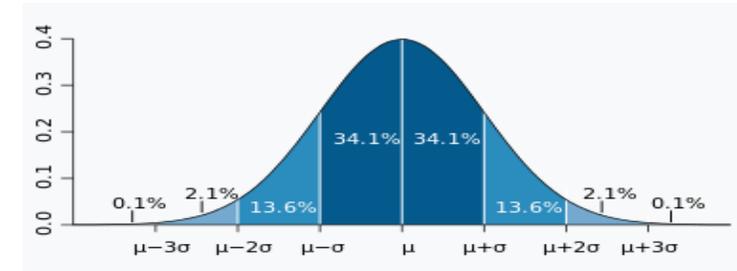
- **Ways to increase Luminosity:**

- 1) increase bunch intensity
- 2) reduce transverse beam sizes at IP
- 3) increase collision frequency

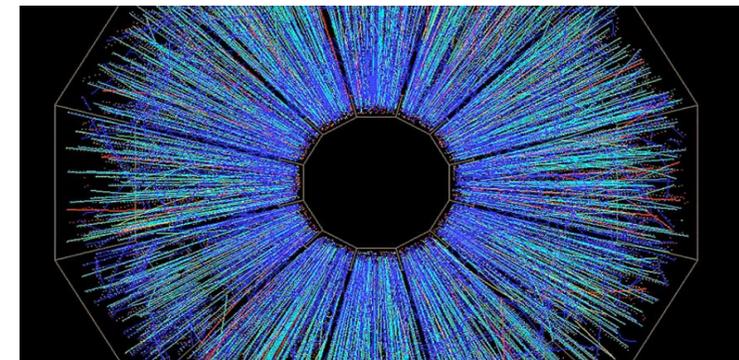
- There are many limits for luminosity increase. Beam-beam Interaction is one of them.



at Interaction Point (IP)



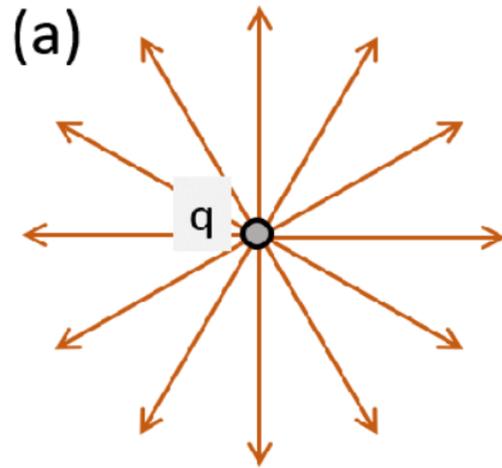
Particle distribution



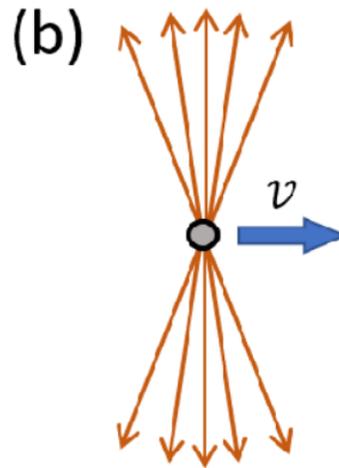
STAR detector in RHIC

Beam-Beam Interaction

- When two bunches of charged particles collide, particles from one bunch will feel the electrical and magnetic fields of the opposite bunch.
- For charged particles with $\gamma \gg 1$, their electrical and magnetic fields will shrink into a thin disc perpendicular to its velocity.

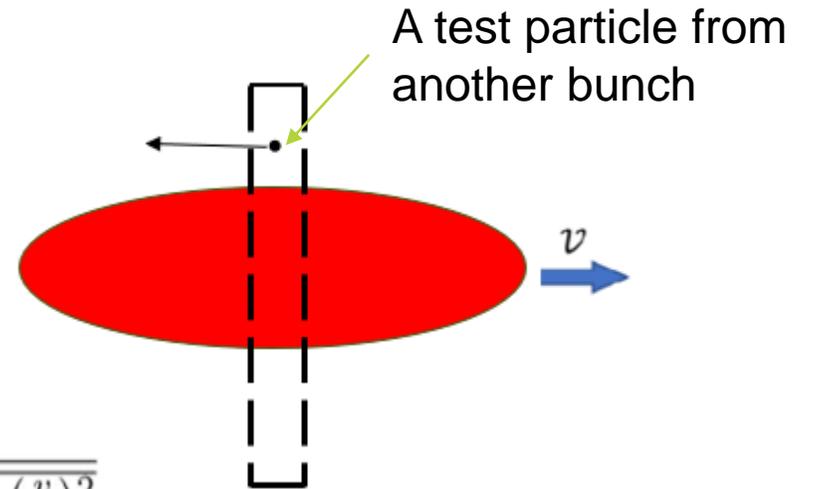


still charge particle



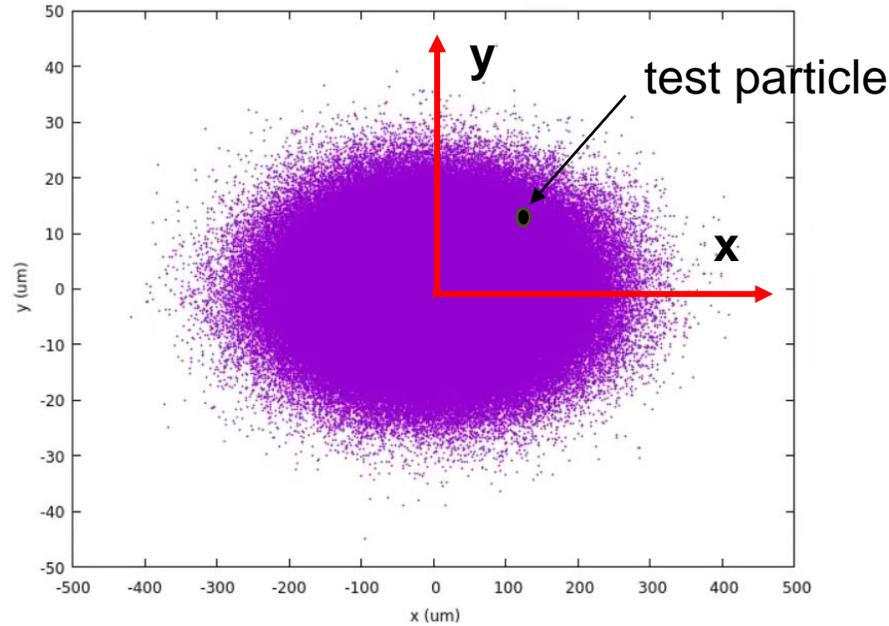
moving charge particle

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$



moving bunch of charge particles

Beam-Beam Force



- **Poisson's Equation**

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \phi(x, y) = -2\pi \rho_c(x, y)$$

$$\vec{E} = -\nabla \phi \quad \mathbf{B} = \vec{\beta} \times \mathbf{E}/c$$

$$\vec{\beta} = \mathbf{v}/c$$

- **Bi-Gaussian Particle Distribution**

$$\rho(x, y) = \frac{ne}{2\pi\sigma_x\sigma_y} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)$$

$$E_x = \frac{ne}{2\epsilon_0\sqrt{2\pi(\sigma_x^2 - \sigma_y^2)}} \operatorname{Im} \left[\operatorname{erf} \left(\frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) - e^{\left(-\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2}\right)} \operatorname{erf} \left(\frac{x\frac{\sigma_y}{\sigma_x} + iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right]$$

$$E_y = \frac{ne}{2\epsilon_0\sqrt{2\pi(\sigma_x^2 - \sigma_y^2)}} \operatorname{Re} \left[\operatorname{erf} \left(\frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) - e^{\left(-\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2}\right)} \operatorname{erf} \left(\frac{x\frac{\sigma_y}{\sigma_x} + iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right]$$

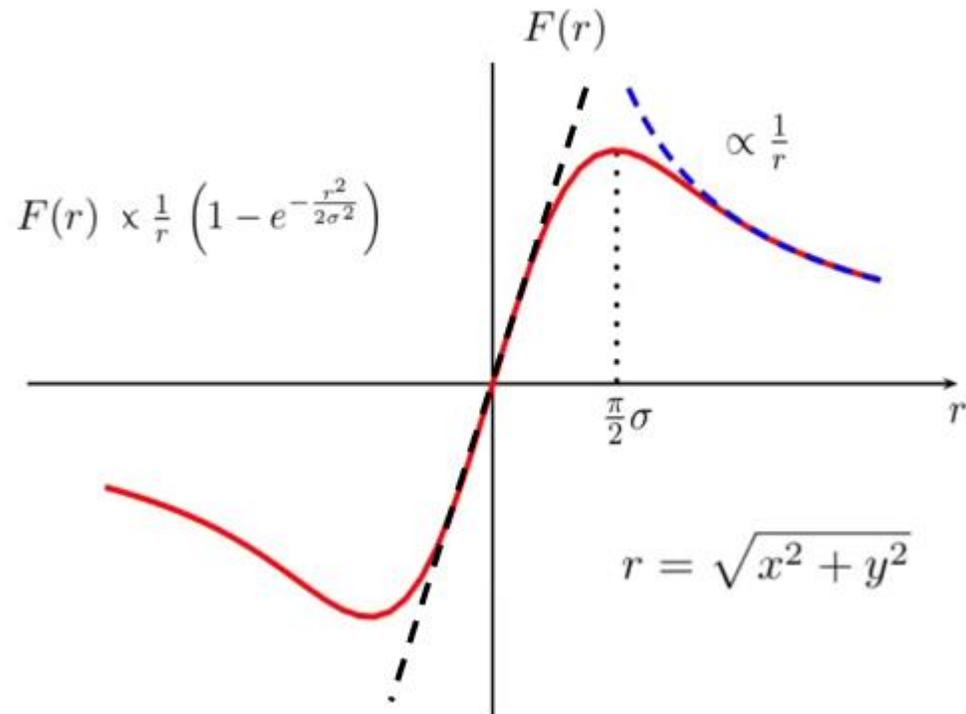
- **Round Gaussian Particle Distribution**

$$E_x = \frac{ne}{2\pi\epsilon_0} \frac{x}{r^2} \left(1 - \frac{r^2}{2\sigma^2} \right)$$

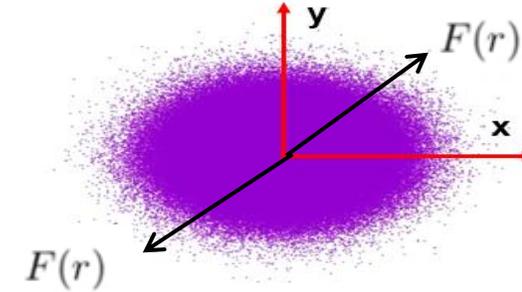
$$E_y = \frac{ne}{2\pi\epsilon_0} \frac{y}{r^2} \left(1 - \frac{r^2}{2\sigma^2} \right)$$

Property of Beam-Beam Force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$



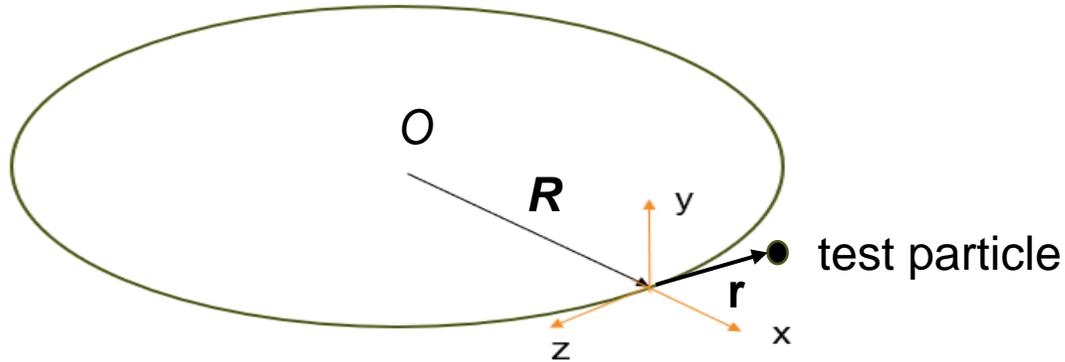
Example: beam-Beam force from a round-Gaussian particle distribution



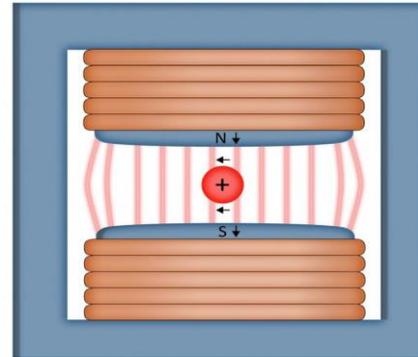
- **Beam-beam force in the direction of r**
(for $\rho_c(x, y) = \rho_c(-x, -y)$ distribution)
- **Beam-beam force proportional to bunch intensity of opposite bunch**
- **Beam-beam force is nonlinear with r**
 - **Linear kick near the bunch core**
 - **$1/r$ decreases for large amplitude**

Linear Particle Motion in Storage Rings

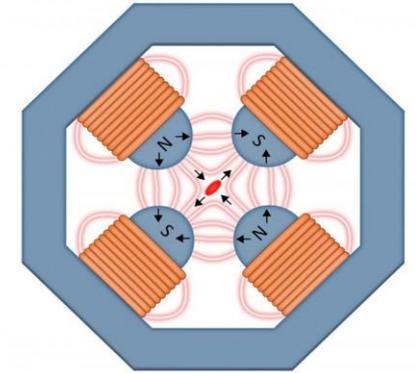
- Coordinate system



Dipole magnet



Quad magnet

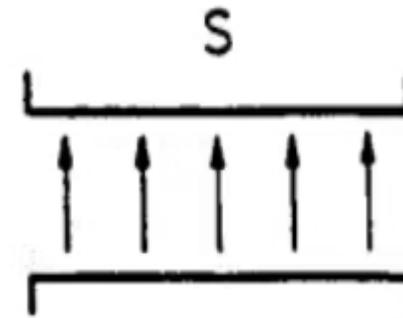


- Linear transverse (Betatron) motion

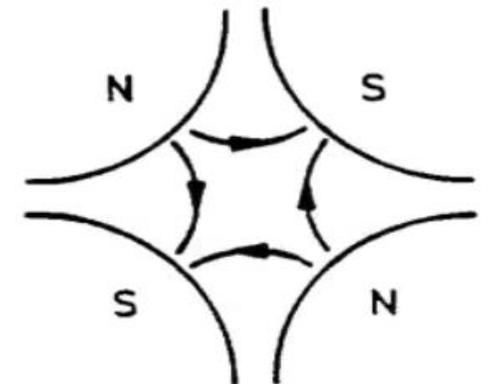
$$\frac{d^2 x(s)}{ds^2} + \left(\frac{1}{\rho^2(s)} + k(s) \right) x(s) = 0$$

$$\frac{d^2 y(s)}{ds^2} - k(s)y(s) = 0$$

Hill's Equation



$$\frac{1}{\rho} = \frac{eB_y}{p_0}$$



$$k = \frac{e}{p_0} \frac{\partial B_y}{\partial x} \Big|_{x=0}$$

Betatron Motion in Storage Rings

- **Betatron motion**

$$x(s) = \sqrt{2J_x\beta_x(s)} \cos\left(\int_0^s \frac{ds}{\beta_x(s)} + \phi_{0,x}\right)$$

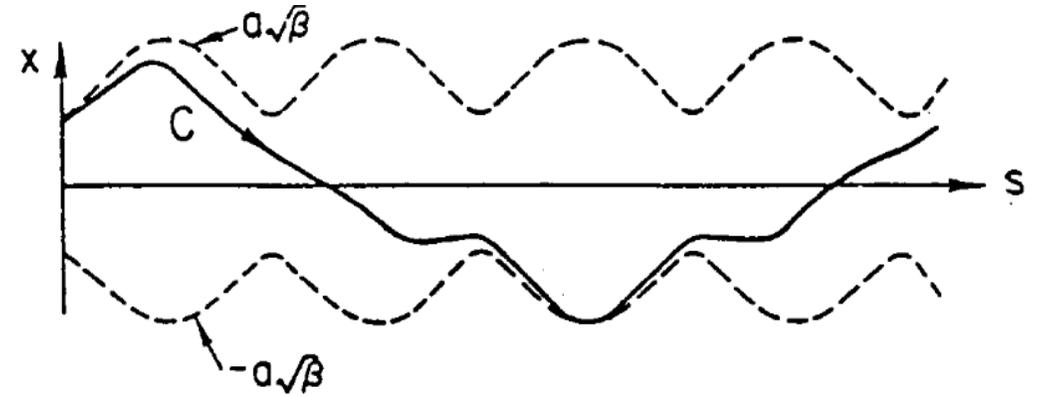
$$y(s) = \sqrt{2J_y\beta_y(s)} \cos\left(\int_0^s \frac{ds}{\beta_y(s)} + \phi_{0,y}\right)$$

- **Betatron tunes (oscillations per turn)**

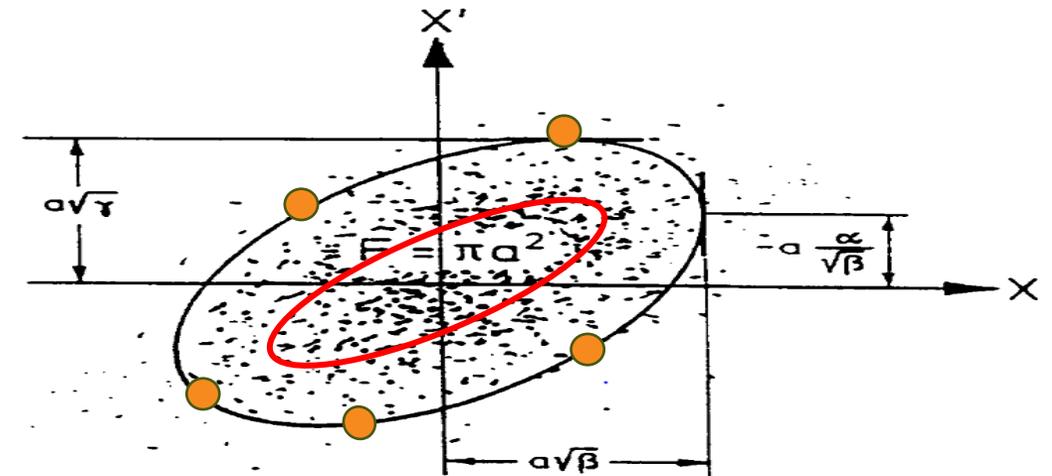
$$Q_x = \frac{1}{2\pi} \oint_0^L \frac{ds}{\beta_x(s)} \quad Q_y = \frac{1}{2\pi} \oint_0^L \frac{ds}{\beta_y(s)}$$

- **Beam emittances (phase space area)**

$$\epsilon_{x,y} = \frac{\sigma_{x,y}^2}{\beta_{x,y}} \quad \sigma_{x,y} = \sqrt{\epsilon_{x,y}\beta_{x,y}}$$



One particle's trajectory along the ring



many particles at one point in the ring

Linear Beam-Beam Tune Shift

- **Perturbed Betatron Motion**

$$x'' + K_x x = \frac{F_x}{\gamma m_0 c^2}$$

$$y'' + K_y y = \frac{F_y}{\gamma m_0 c^2}$$

- **Linear beam-beam tune shift**

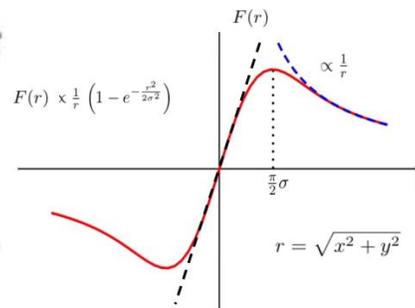
$$\Delta Q_x = \frac{N r_0 \beta_x^*}{2\pi \gamma \sigma_x (\sigma_x + \sigma_y)}$$

$$\Delta Q_y = \frac{N r_0 \beta_y^*}{2\pi \gamma \sigma_y (\sigma_x + \sigma_y)}$$

- **Linear beam-beam kick (for test particles near the bunch core)**

$$\Delta x' = \frac{2N r_0}{\gamma \sigma_x (\sigma_x + \sigma_y)} x$$

$$\Delta y' = \frac{2N r_0}{\gamma \sigma_y (\sigma_x + \sigma_y)} y$$

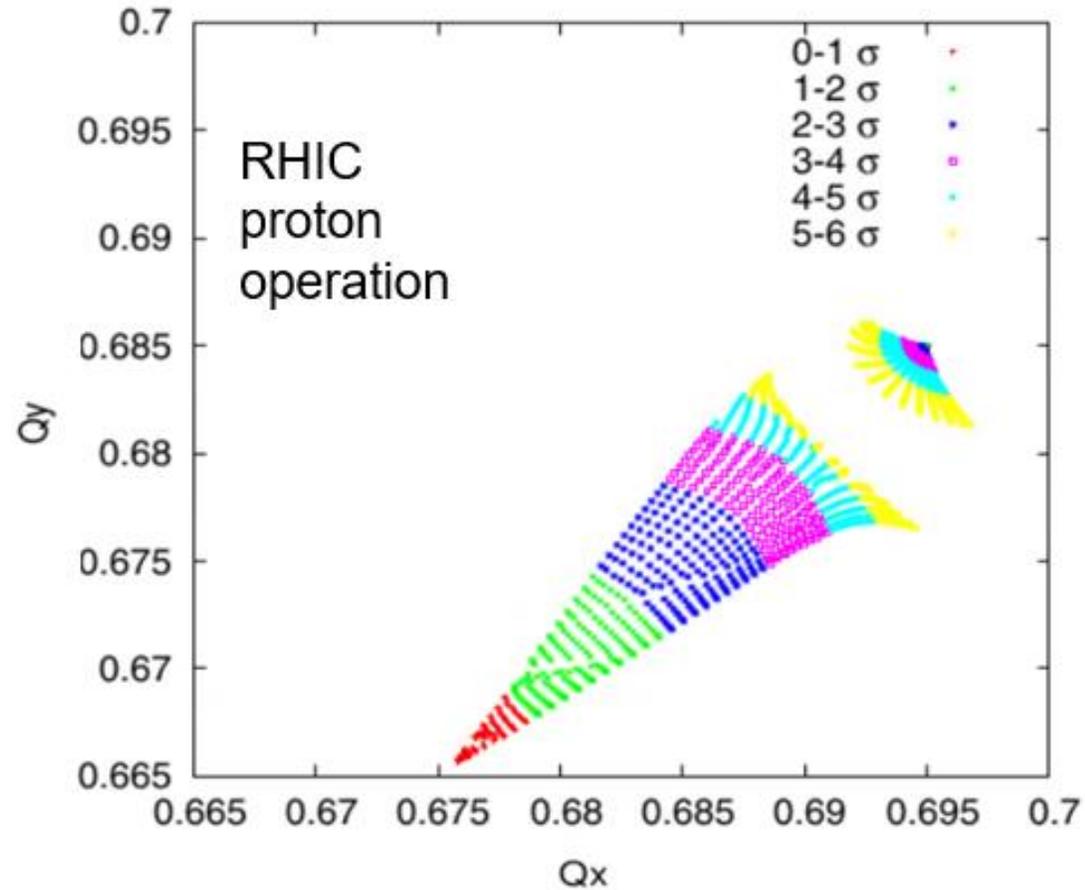


- **Beam-Beam Parameter**

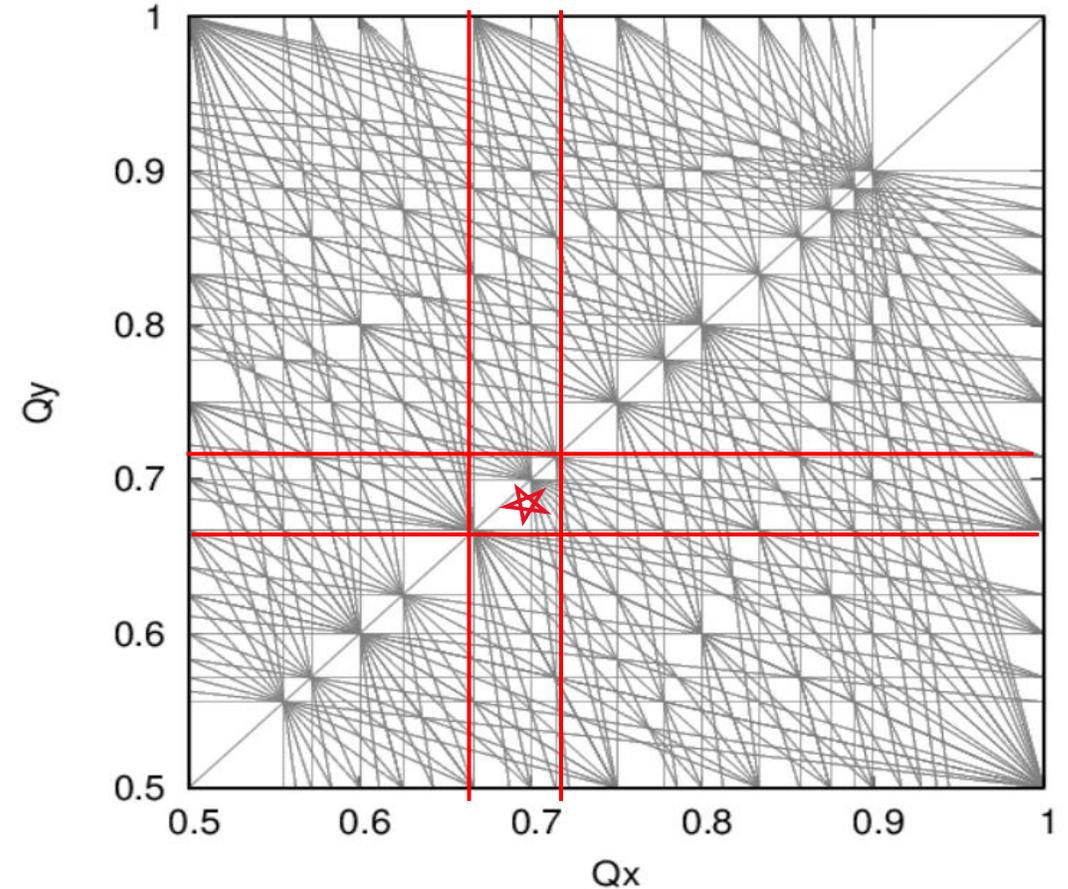
$$\xi_{x,y} = \frac{N r_0 \beta_{x,y}^*}{2\pi \gamma \sigma_{x,y} (\sigma_x + \sigma_y)}$$

A larger bunch intensity and smaller beam transverse sizes at interaction point (IP) will give a larger beam-beam parameter.

Nonlinear Beam-Beam Tune Shift / Tune Spread



Tune footprints with beam-beam interaction
Design tunes are (28.695, 29.685).



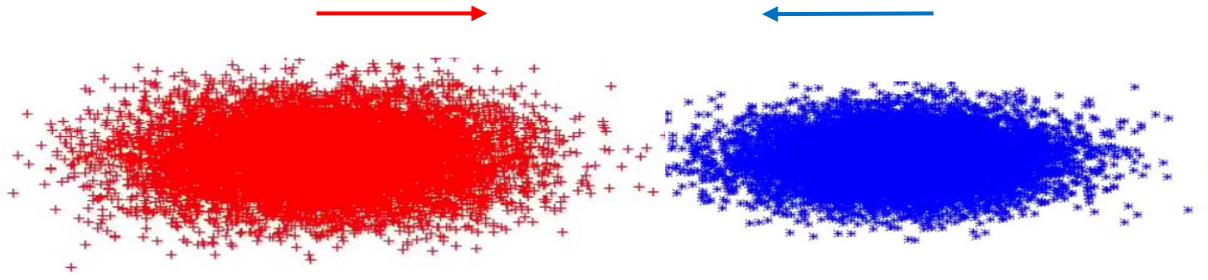
$m Q_x + n Q_y = p$, m, n, p are integers
Up to 10th order: $|m| + |n| < 11$ in the plot

Beam-Beam Effects

- Beam-beam tune shift
 - Beam-beam tune spread
 - Dynamic β -effect, dynamic emittance
 - **Nonlinear motion**, 2-d and 3-d coupled nonlinear resonances,
 - Resonance overlapping, chaotic motion, dynamic aperture
 - Coherent beam-beam motion, coherent beam-beam instability
 - Single interaction point, multiple interaction points per turn
 - **Beam-beam limit** (maximum beam-beam parameter)
 - for hadron colliders: beam-beam limit < 0.03
 - for electron colliders: beam-beam limit < 0.15
 - Head-on collision, long-range beam-beam,
 - **Crossing angle collision**, crab cavities, crab waist scheme
 - Head-on beam-beam compensation, long-range beam-beam compensation
 - Interplay between beam-beam and wake field, space charge, polarization
 -
 - **Particle loss, bad beam lifetime**
 - **Emittance increase, beam-beam tail, emittance blowup**
 - **Luminosity reduction, bad luminosity lifetime**
- } **observations**

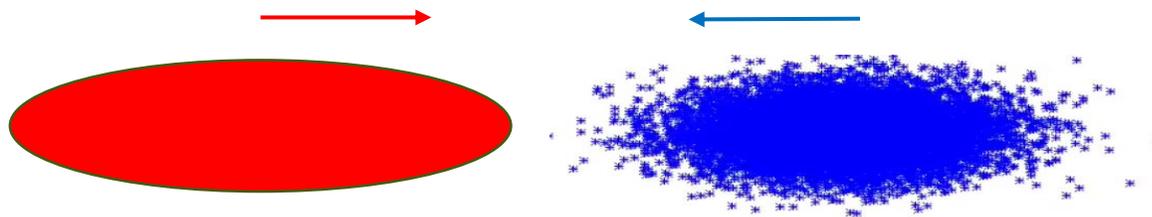
Numerical Simulation Models and Tools

- **Strong-strong simulation model**



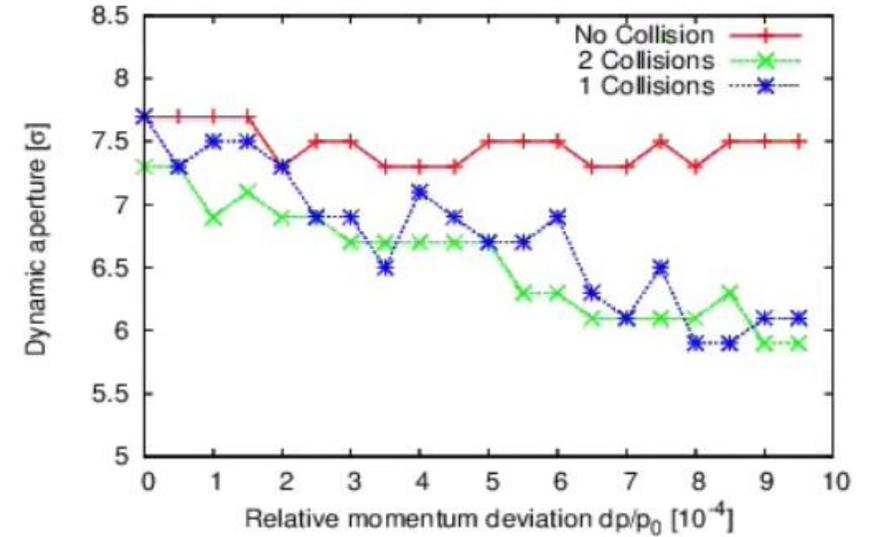
Both bunches are represented by ~ a million macro-particles. Subject to numerical noises.

- **Weak-Strong simulation model**



Strong bunch is rigid while weak bunch is represented by macro-particles. Not consistent.

- **Dynamic aperture calculation**



- **Other analysis tools**

- Frequency Map Analysis (FMA)
- Action diffusion
- Resonance driving terms (RDT)

Outline

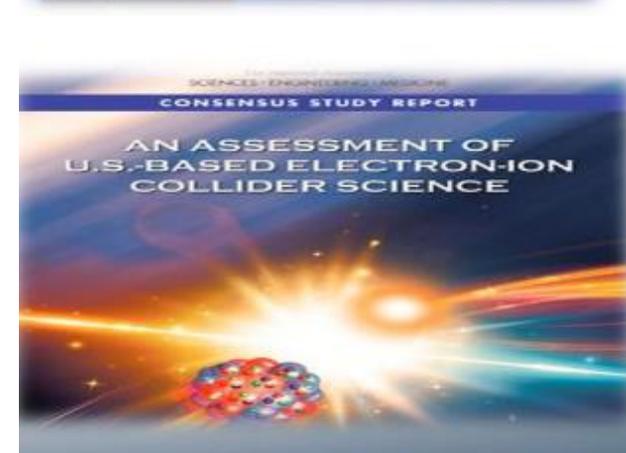
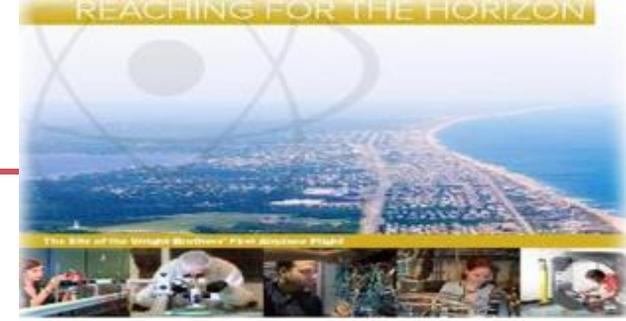
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Design Goals of Electron-Ion Collider

Project Design Goals:

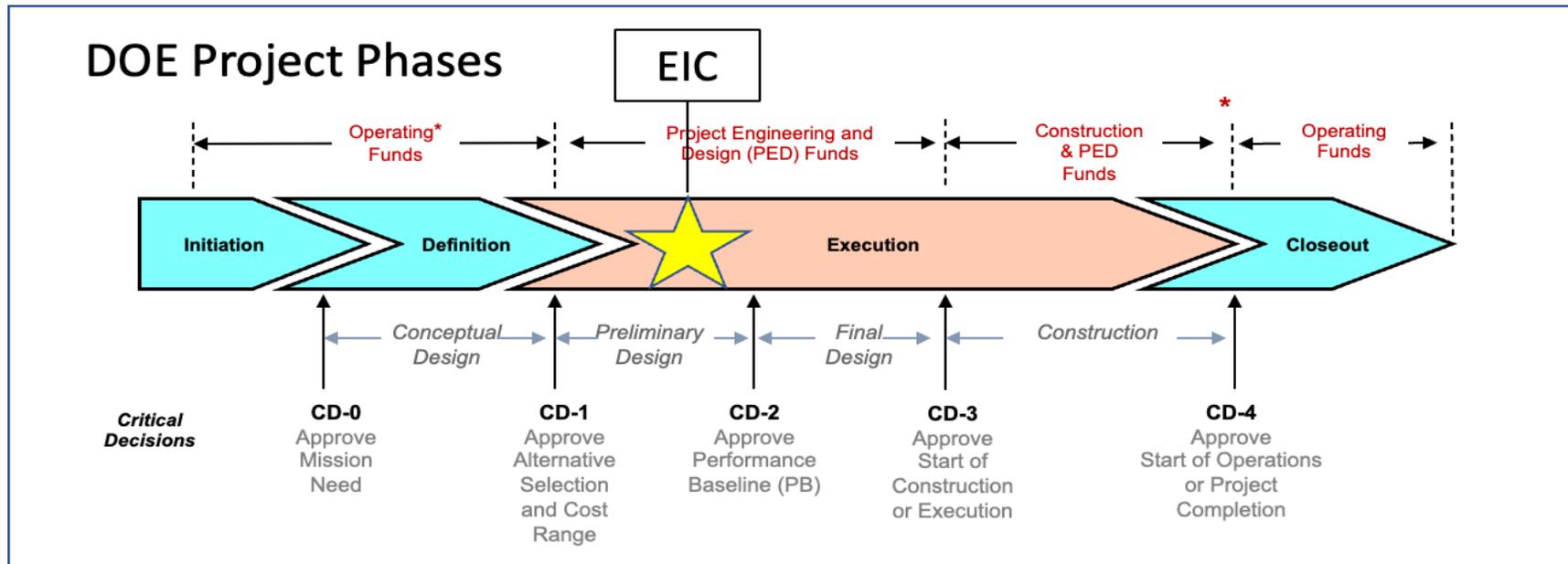
- High Luminosity: $L = 10^{33} - 10^{34} \text{cm}^{-2}\text{sec}^{-1}$, 10 – 100 fb⁻¹/year
- Highly Polarized Beams: 70%
- Large Center of Mass Energy Range: $E_{\text{cm}} = 20 - 140 \text{ GeV}$
- Large Ion Species Range: protons – Uranium
- Large Detector Acceptance and Good Background Conditions
- Accommodate a Second Interaction Region (IR)

Conceptual design scope and expected performance meet or exceed the Nuclear Science Advisory Committee (NSAC) Long Range Plan (2015) and the EIC White Paper requirements endorsed by the National Academy of Sciences (2018). 2023 NSAC Long Range Plan reaffirmed the exceptional high priority of EIC.



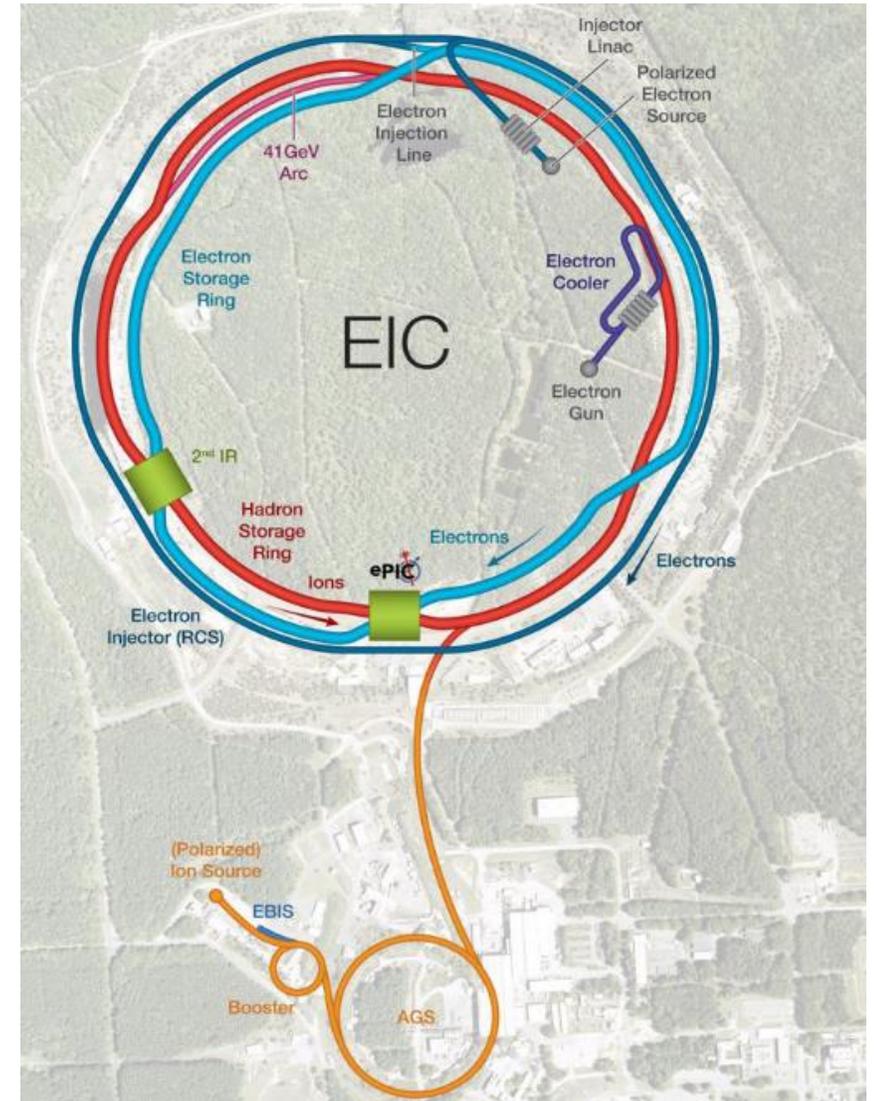
EIC Project Critical Decisions and Plans

CD-0, Mission Need Approved	December 2019
DOE Site Selection Announced	January 2020
CD-1, Alternative Selection and Cost Range, Approved	June 2021
CD-3A, Long Lead Procurement Approval	January 2024
CD-3B, Long Lead Procurement Approval	October 2024
CD-2/3, Performance Baseline/Construction Start	April 2025



Accelerator Complex of Electron-Ion Collider

- Based on **existing, well maintained, well performing RHIC**
- **Hadron storage Ring (RHIC Rings) 40-275 GeV**
 - Superconducting magnets (existing)
 - 1160 bunches, 1A beam current (3x RHIC)
 - Bright vertical beam emittance 1.5 nm (“flat beams”)
 - Strong cooling (coherent electron cooling)
- **Electron Storage Ring 2.5–18 GeV**
 - Large beam current, 2.5 A, 9 MW S.R. power, S.C. RF
 - Need to inject polarized bunches
- **Electron rapid cycling synchrotron, 1Hz, (0.4- 18) GeV**
 - Spin transparent due to high quasi periodicity
- **High luminosity Interaction Region(s)**
 - Superconducting final focus magnets
 - 25 mrad crossing angle with crab cavities
 - Spin Rotators (longitudinal spin)
 - Forward hadron instrumentation



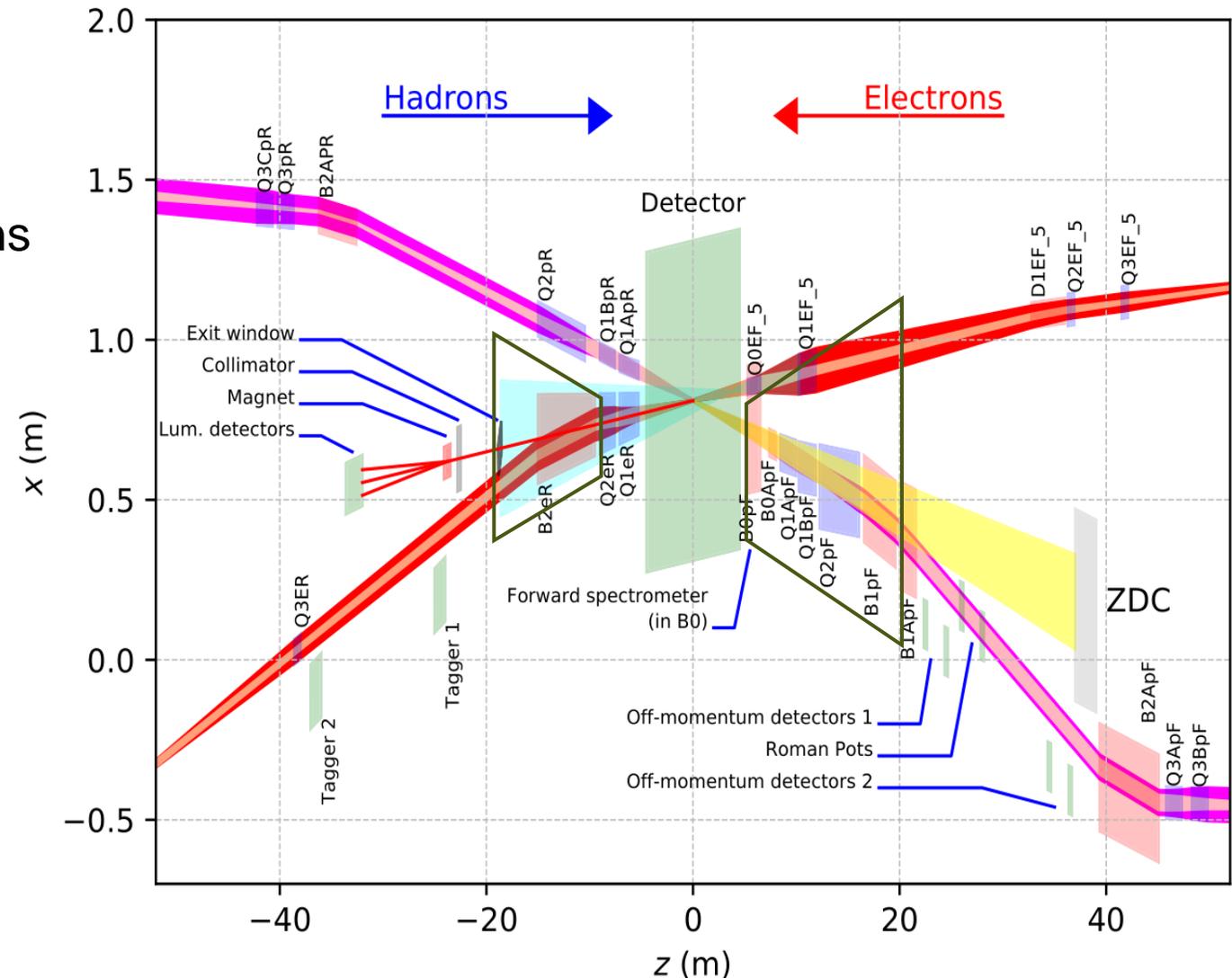
EIC Interaction Region Layout

High luminosity:

- 25 mrad crossing angle
- Small β^* for high luminosity with limited IR chromaticity contributions
- Large final focus quadrupole aperture

Machine Detector Interface

- Large detector acceptance
- Forward spectrometer
- No magnets within - 4.5 / +5 m from IP
- Space for luminosity detector, neutron detector, “Roman Pots”



Outline

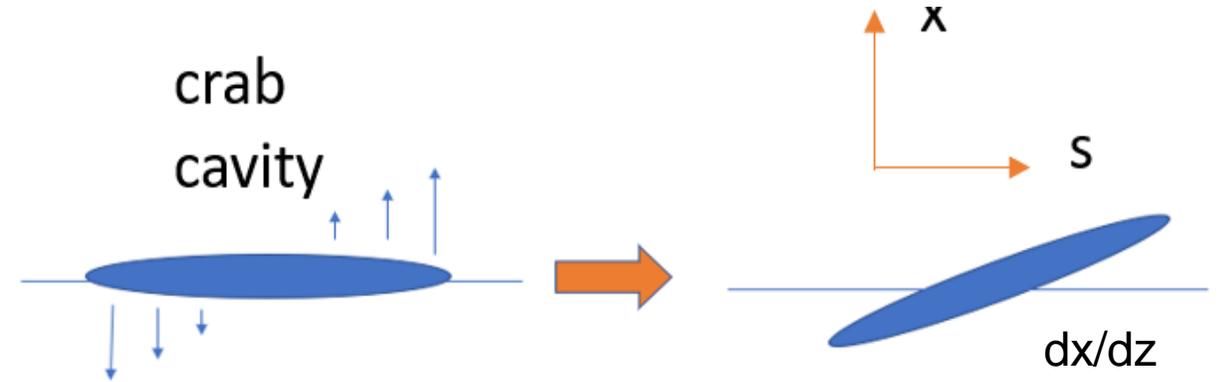
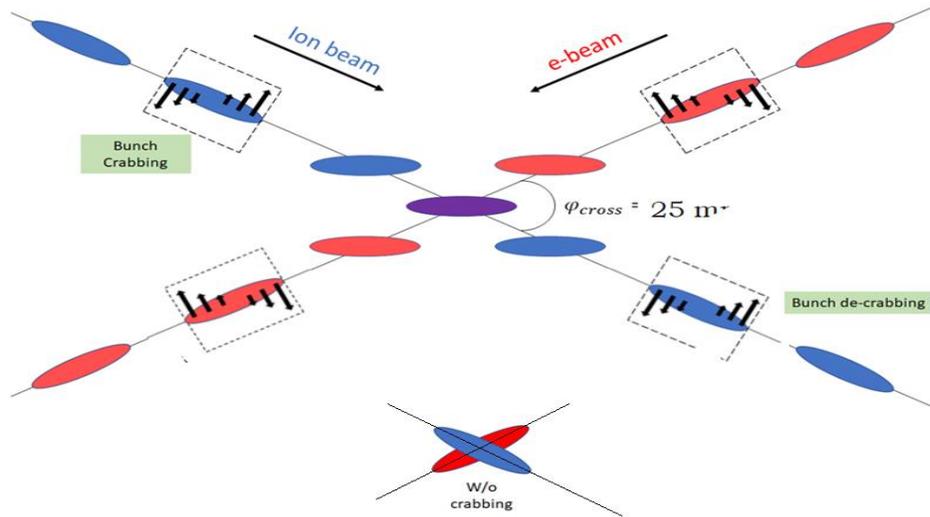
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Design Parameters

For the highest luminosity collision mode

Parameter	proton	electron
Ring circumference [m]	3833.8451	
Particle energy [GeV]	275	10
Lorentz energy factor γ	293.1	19569.5
Bunch population [10^{11}]	0.688	1.72
RMS emittance (H,V) [nm]	(11.3, 1.0)	(20.0, 1.3)
β^* at IP (H, V) [cm]	(80, 7.2)	(45, 5.6)
RMS bunch size σ^* at IP (H, V) [μm]	(95, 8.5)	
RMS bunch length σ_l at IP [cm]	6	0.7
Beam-beam parameters (H, V)	(0.012, 0.012)	(0.072, 0.1)
RMS energy spread [10^{-4}]	6.8	5.8
Transverse tunes (H,V)	(29.228, 30.210)	(51.08, 48.14)
Synchrotron tune	0.01	0.069
Longitudinal radiation damping time [turn]	-	2000
Transverse radiation damping time [turn]	-	4000
Luminosity [$10^{34}\text{cm}^{-2}\text{s}^{-1}$]	1.0	

Crossing Angle Collision With Crab Cavities



Schematic plot of crab cavity kick (force)

Crab cavity's Hamiltonian:

$$H_{\text{crab}} = \frac{qV}{E_s} \sin\left(\phi_s + \frac{\omega z}{c}\right)x,$$

$$\Delta p_x = -\frac{\partial H_{\text{crab}}}{\partial x} = -\frac{qV}{E_s} \sin\left(\phi_s + \frac{\omega z}{c}\right),$$

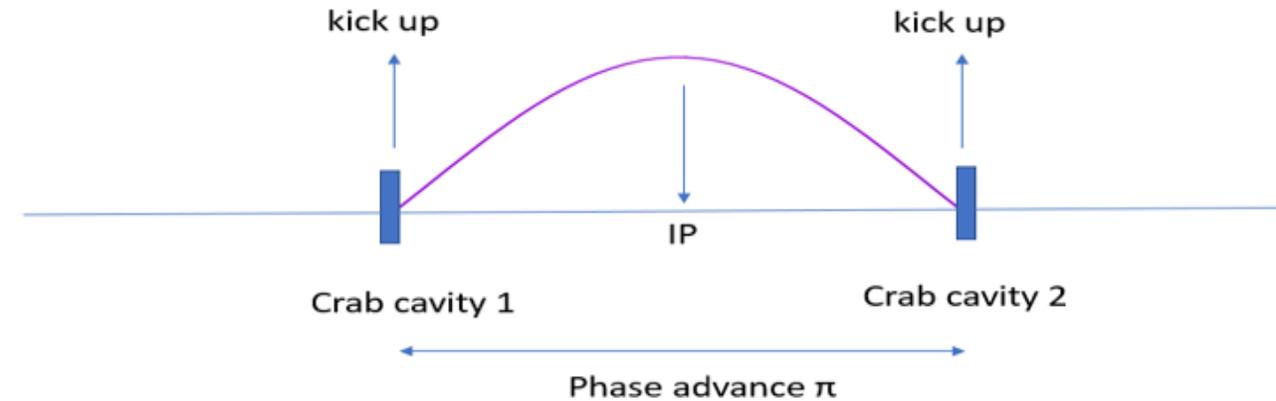
$$\Delta p_z = -\frac{\partial H_{\text{crab}}}{\partial z} = -\frac{qV}{E_s} \cos\left(\phi_s + \frac{\omega z}{c}\right) \frac{\omega}{c} x.$$

linearizing \rightarrow

$$\Delta p_x = -K_c z$$

$$\Delta \delta = -K_c x$$

$$k_c = -\frac{qV\omega}{E_s c}$$

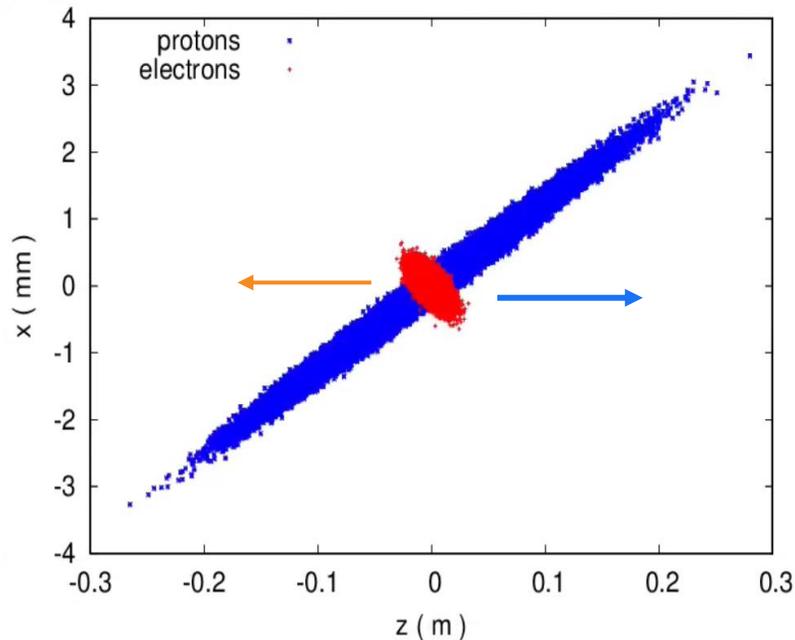


Ideal phase advance between crab cavities

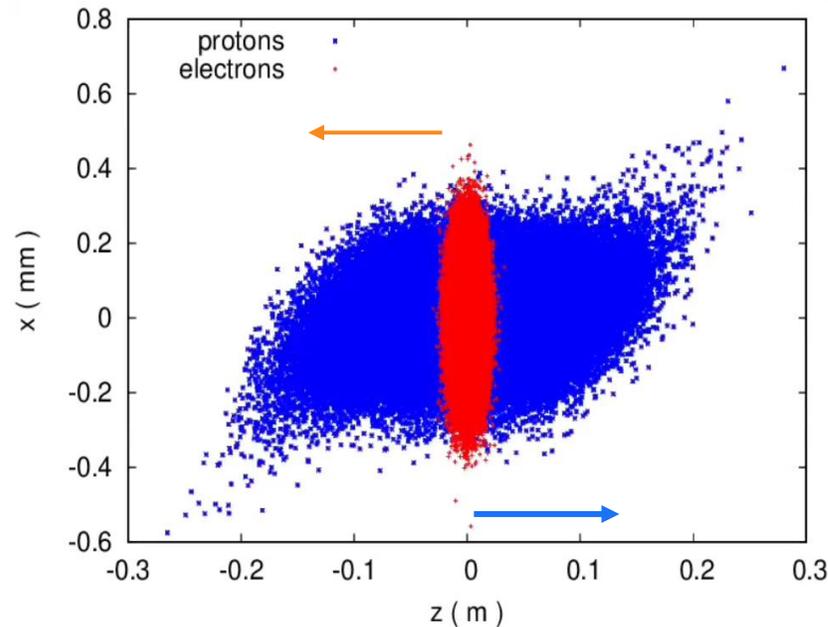
EIC Crab Cavities

- **ESR:** 394 MHz, 2 cells in 2 cryomodules each side, maximum 3.5 MV totally.
- **HSR:** 197 MHz, 4 cells in 2 cryomodules each side, maximum 35 MV totally.
Second harmonic crab cavities 394 MHz (~5 MV) are needed to further straighten proton bunch shape and to improve proton lifetime.

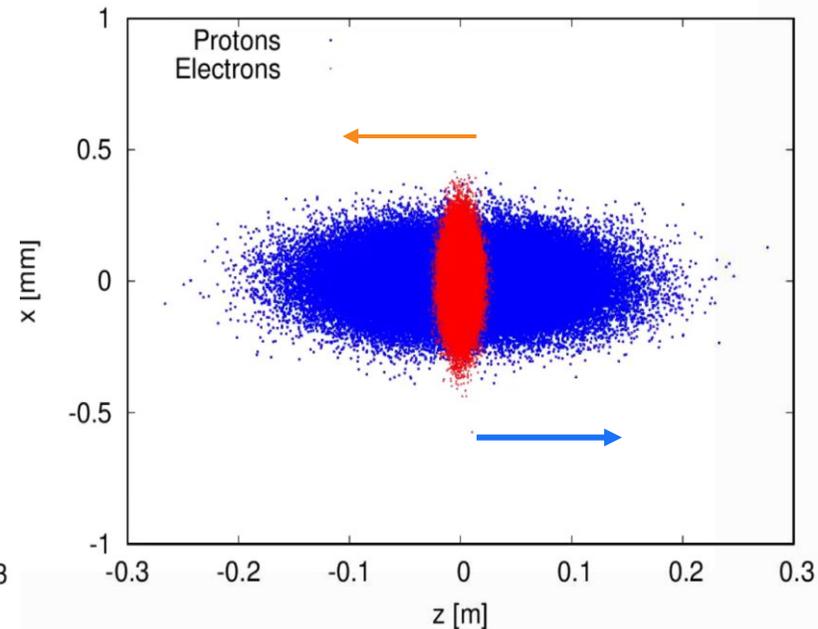
No crab cavity in both rings



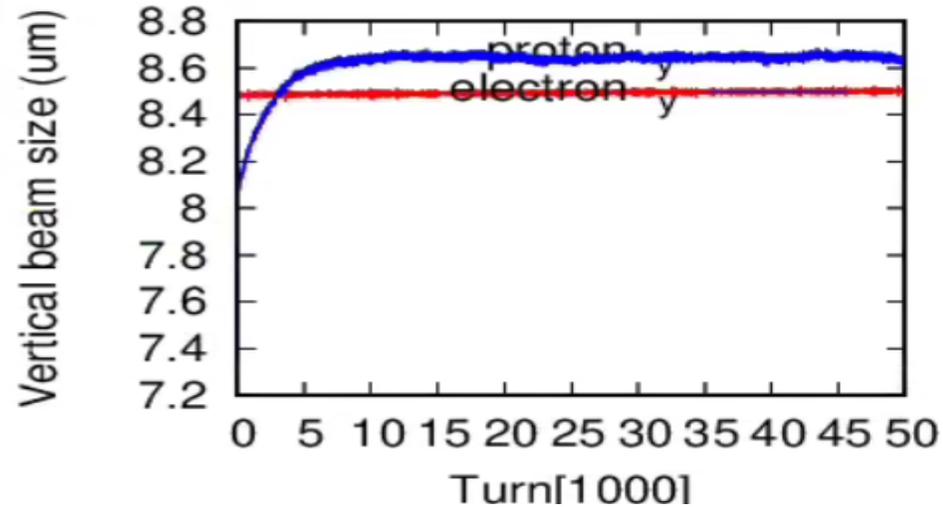
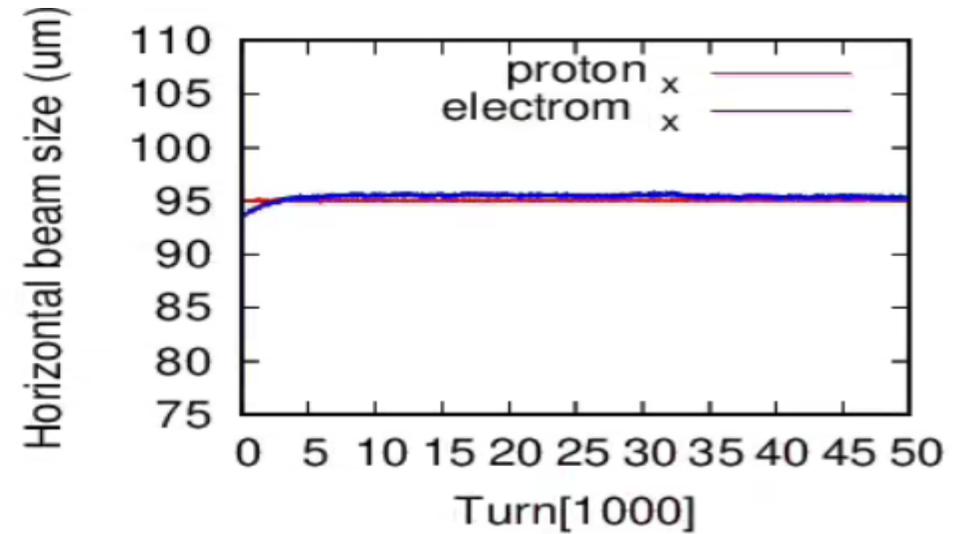
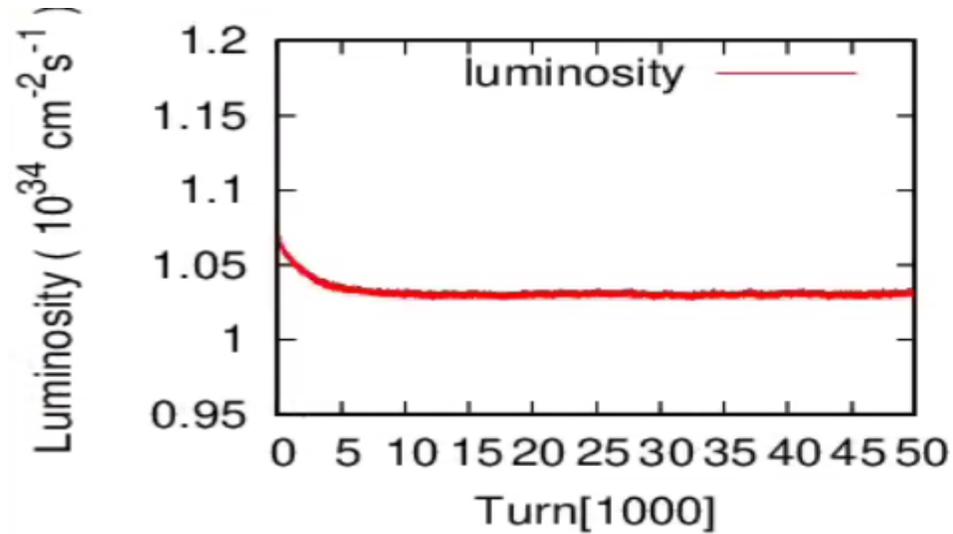
Only 197MHz crab cavities in HSR



With harmonic crab cavities in HSR



Luminosity and Beam Sizes from Simulation



For the highest peak luminosity collision mode between 275GeV proton and 10 GeV electron beams.

Strong-strong beam-beam simulation

Challenges in EIC BB Interaction

- **High beam-beam parameters**

Proton BB parameter ~ 0.015 , Electron BB parameter ~ 0.1
combination not demonstrated in early electron-proton collider

- **Large crossing angle**

full crossing angle is 25 mrad in IR6

- **Crab cavities used in both rings**

crab cavities had been used in KEK-B, not used in hadron collider yet
crab dispersion leakage, interference between detector solenoid and crab cavities
crab cavity multipoles, voltage and phase noises of crab cavities

- **Flat beam at IP and large transverse emittance ratio**

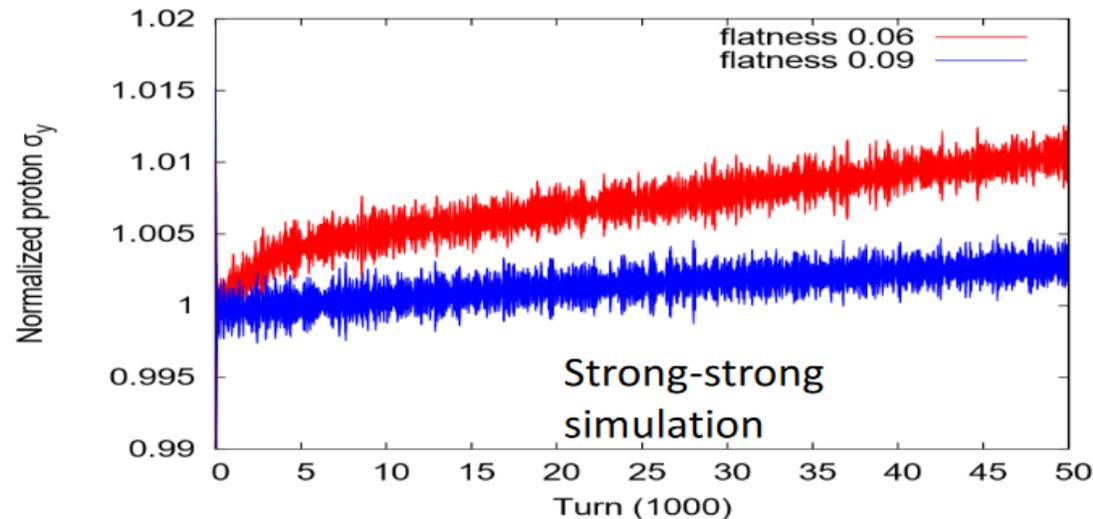
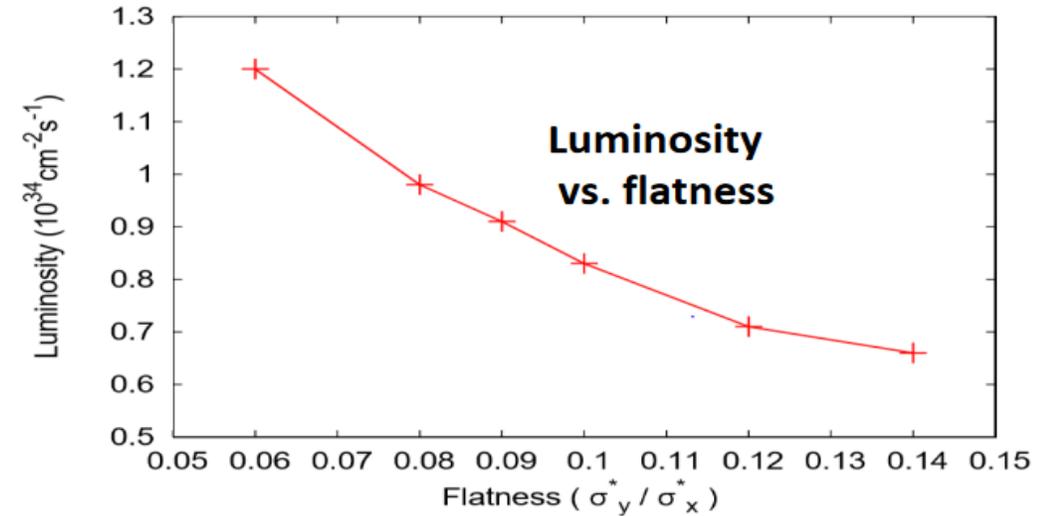
need very strict coupling control, vulnerable vertical emittance growth with BB

- **Other concerns**

near-integer electron tunes \rightarrow pinch effect \rightarrow larger proton BB parameter
synchro-betatron resonances with large crossing angle and large synchrotron tunes

Flatness & Beam-Beam Performance

- **Key design parameters for BB performance:** BB parameter, flatness, working points, etc.
- **Flatness (σ_y^* / σ_x^*) at IP affects overall BB performance: DA and emittance growth.**
- Flatness 0.09 was chosen for the EIC e-p collision to achieve the maximum design peak luminosity $1 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ and to maintain a relatively low proton emittance growth rate.



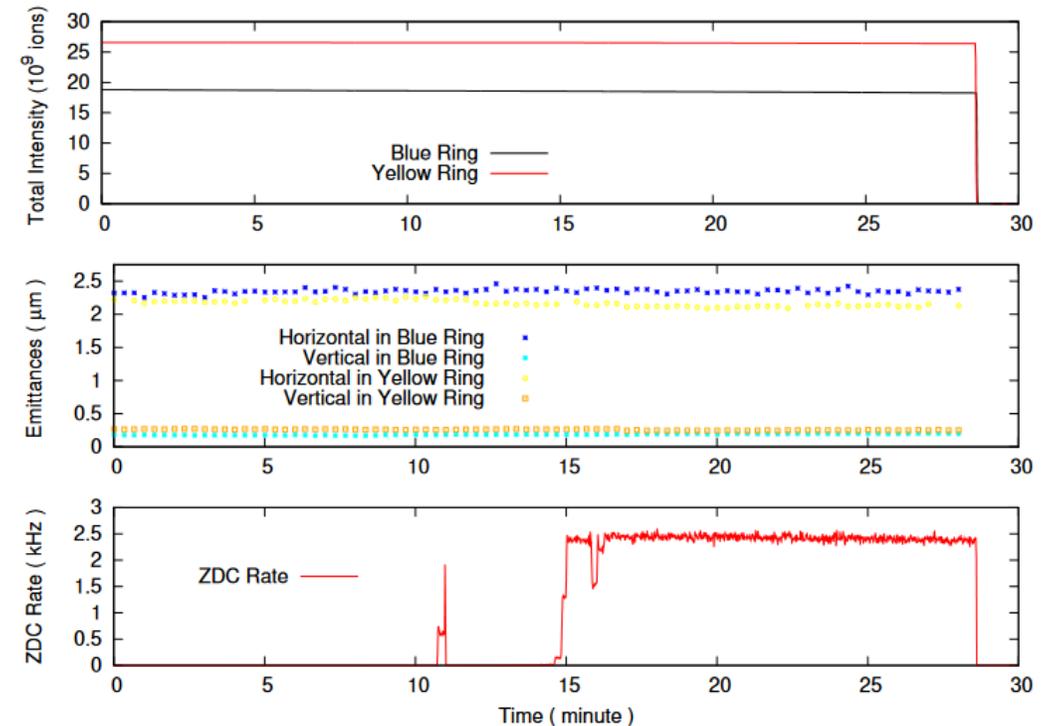
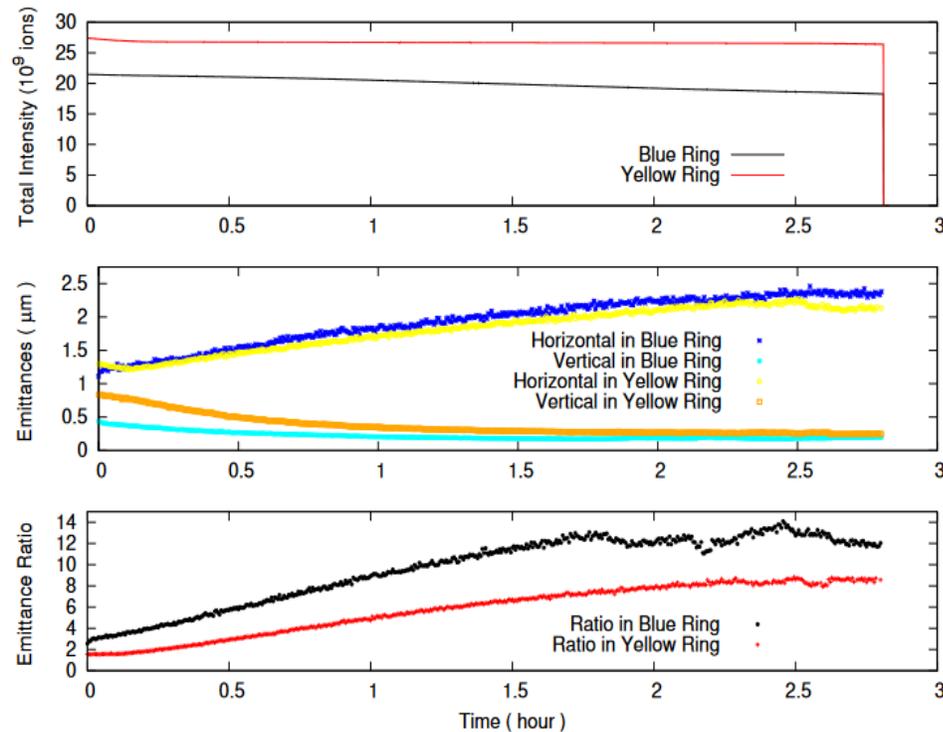
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Weak-strong simulation

flatness	ID	proton beta*x,y [cm]	proton horizontal growth rate [%/hour]	proton vertical growth rate [%/hour]
0.06	43-47	90/5.4	-0.68+/-1.10	14.57+/-7.1
0.08	33-37	90/7.2	-0.52+/-0.37	2.6 +/-1.9
0.09	49-53	90/8.1	0.16+/-0.94	2.7+/-3.1
0.10	28-32	90/9.0	0.09+/-0.86	1.2+/-2.6
0.12	one seed	90/10.9	0.05	0.87
0.14	59-63	90/12.6	-1.2+/-2.5	0.27+/-8.9
0.09	91-95	80/7.2	0.11+/-0.75	3.3+/-3.2

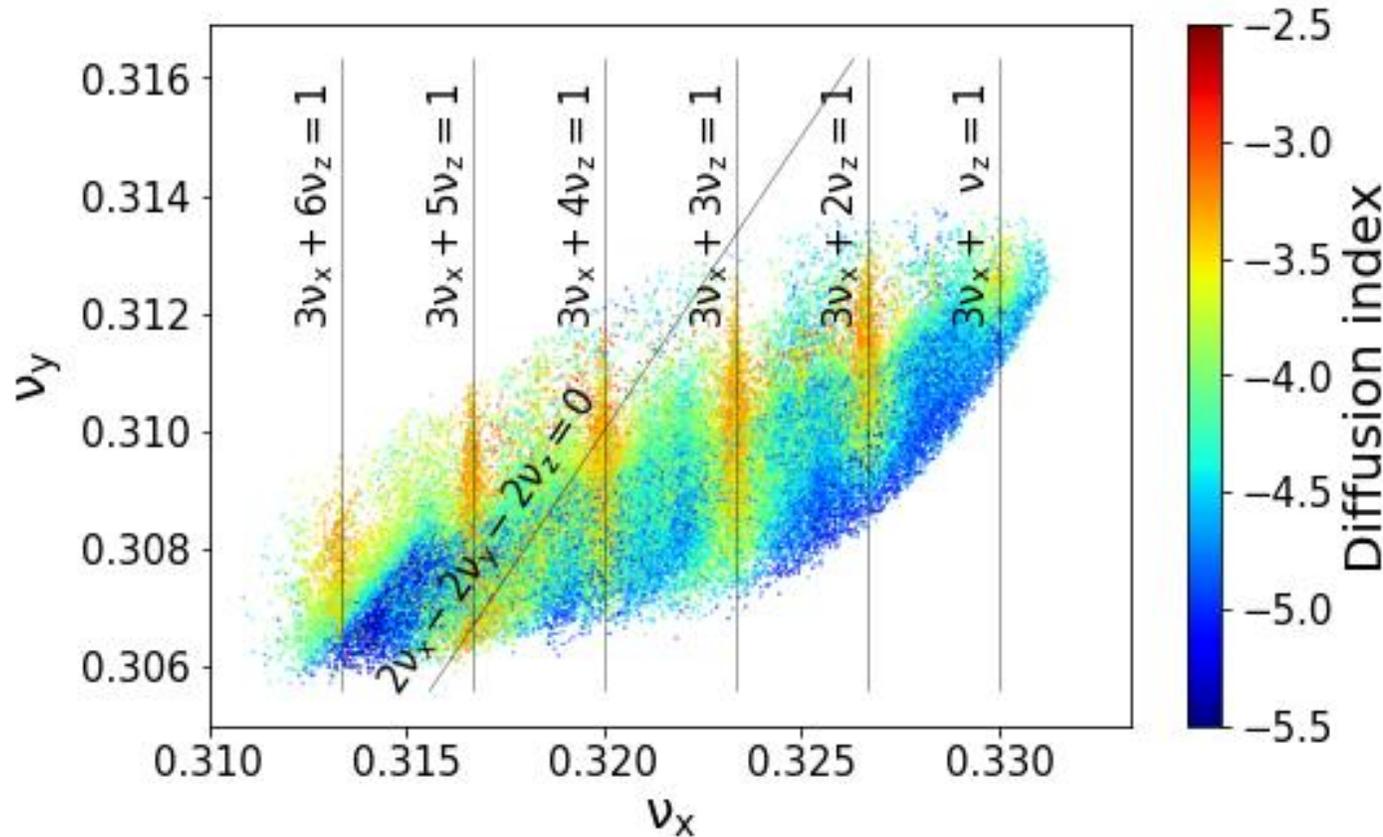
Demonstration of Flat Beams in RHIC

- We experimentally demonstrated 11:1 transverse ratio in RHIC with gold ion beam at 100 GeV/nucleon **with vertical stochastic cooling and fine decoupling.**
- We also collided flat beams with transverse beam size ratio about 3:1.
- Another experiment in preparation to get this emittance ratio at 24 GeV and then accelerate it to 100 GeV.



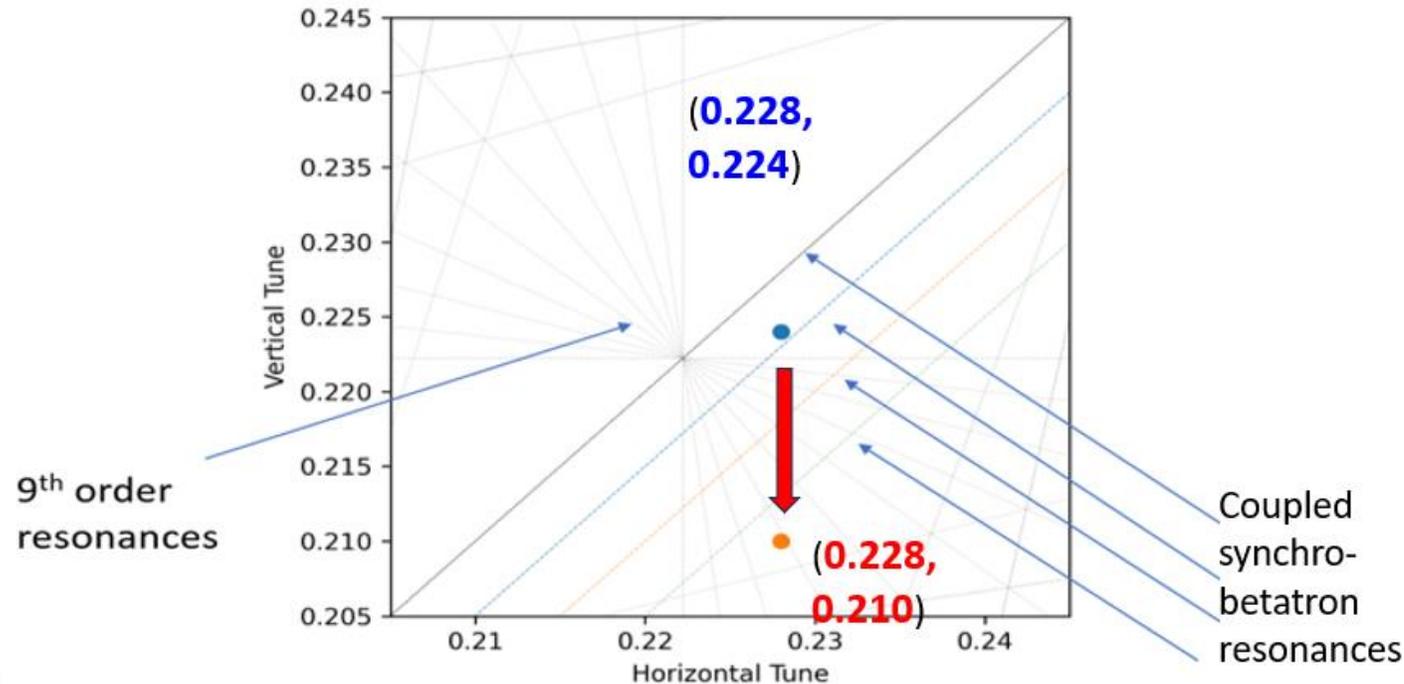
Synchro-Betatron Resonances in HSR

Frequency map analysis (FMA) with working point (0.310, 0.305)



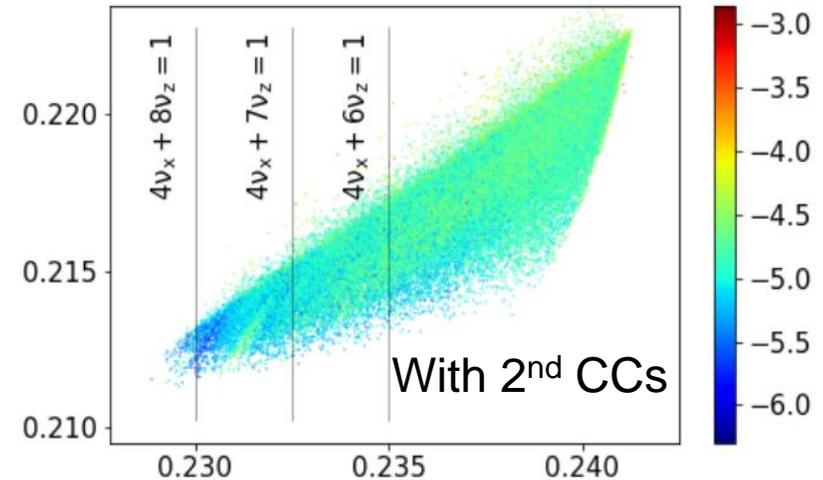
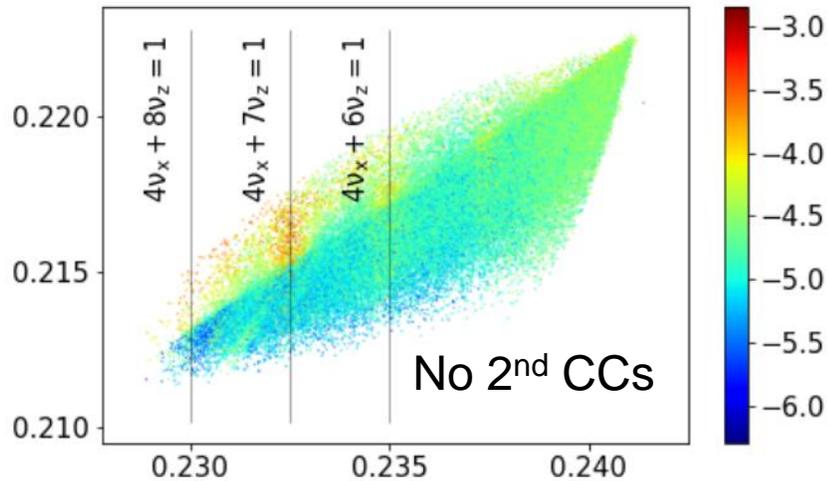
- Synchro-betatron resonances have been observed in many EIC BB simulation studies.
- Two kinds of synchro-betatron resonances identified by FMA:
 - 1) $m \cdot Q_x + p \cdot Q_s$
 - 2) $2 \cdot Q_x - 2 \cdot Q_y + p \cdot Q_s$
- Mitigation measures:
 - 1) working point optimization,
 - 2) second harmonic crab cavities.

Proton Working Point Optimization

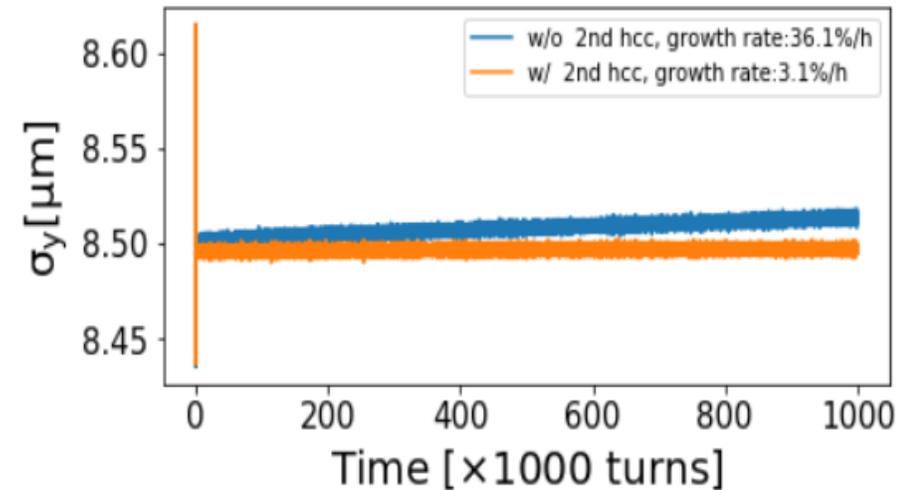


- Moving original **(0.310, 0.305)** down to **(0.228, 0.224)**: 1st kind of resonances are changed from $3*Q_x + p*Q_s$ to $4*Q_x + p*Q_s$.
- Increasing tune split from **(0.228, 0.224)** to **(0.228, 0.210)**: 2nd kind of resonances $2*Q_x - 2*Q_y + p*Q_s$ are excited with a higher order p .

Second Harmonic Crab Cavities in HSR



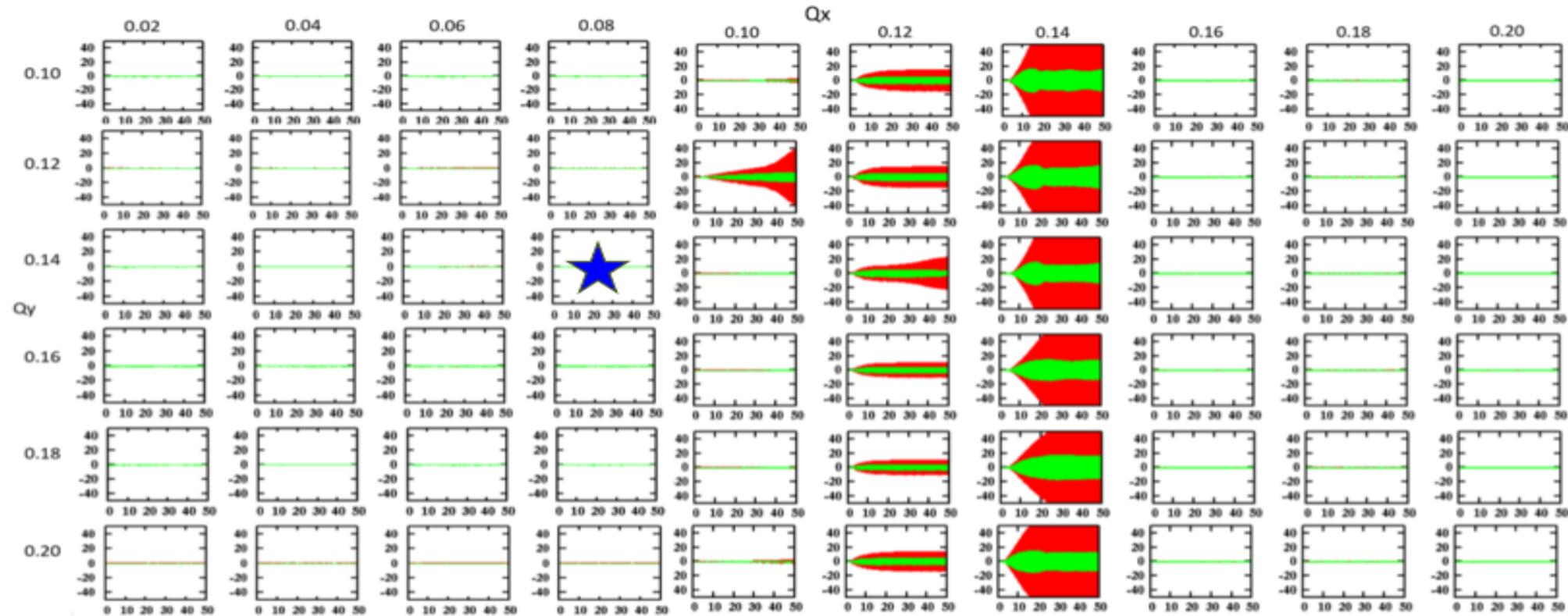
- **BB simulations show that second harmonic crab cavities improve proton vertical emittance growth and dynamic aperture.**
- Second harmonic crab cavities are included in the baseline HSR design.



Weak-strong simulation

Coherent Beam-Beam Instability

- Coherent BB instability was only observed during electron tune scan when electron beam's horizontal tune is between 0.1 and 0.14, which is caused by coupling resonance $m \cdot Q_{x,p} + n \cdot Q_{x,e}$.
- ESR design tunes are (0.08, 0.14), the blue star shown in the plot.



$\langle X_p \rangle$ and $\langle X_e \rangle$ in electron tune scan

Machine Imperfection and Noises

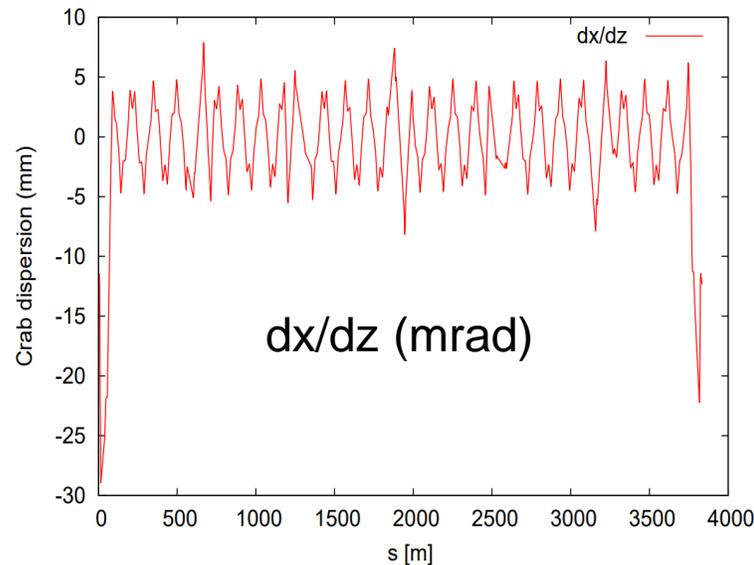
To achieve the peak design luminosity and sufficient beam-beam lifetime, we need to have a very good control of optics and machine imperfections and keep them below tolerances.

- **Optics Imperfections:** Twiss parameters at IP and crab cavities, phase advances between IP and crab cavities, crabbing bump closure, detector solenoid effect, vertical crab dispersion at IP, crab dispersion leakage, etc.
- **Machine Imperfections:** misalignment and roll errors of magnets, magnetic nonlinear field errors, multipoles in crab cavities, nonlinear fields in arc dipoles (important for radially shifted design orbits), etc.
- **Noises:** phase and voltage noises of crab cavities, power supply current ripples, 10Hz orbit oscillations due to cryo-flows (observed in RHIC), RF phase noises, etc.

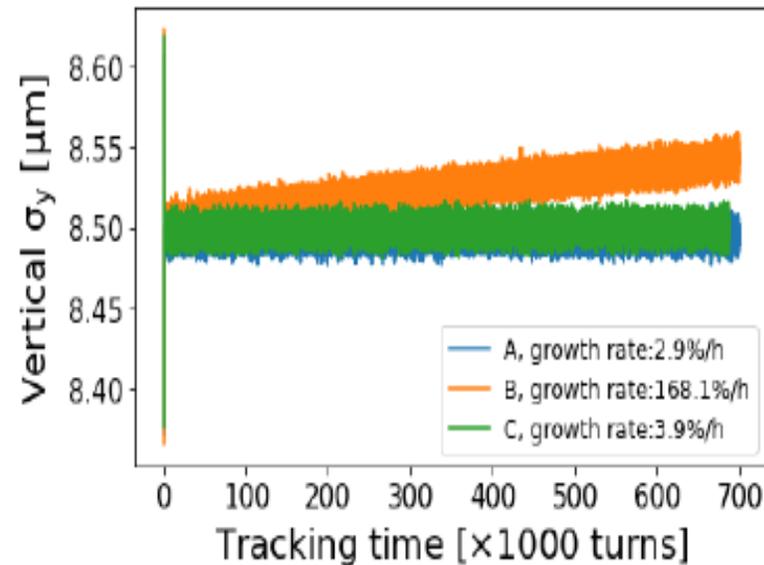
Crabbing Bump Closure in HSR

- **Phase advance between crab cavities: 175 degrees, 5 degrees off 180 degrees.**
- The beam-beam performance can be restored by adjusting crab cavity voltages.
- Dynamic aperture (DA) studies show no significant difference between unclosed and artificially closed crabbing bumps. Mostly likely, the HSR will live with the crabbing leakage.

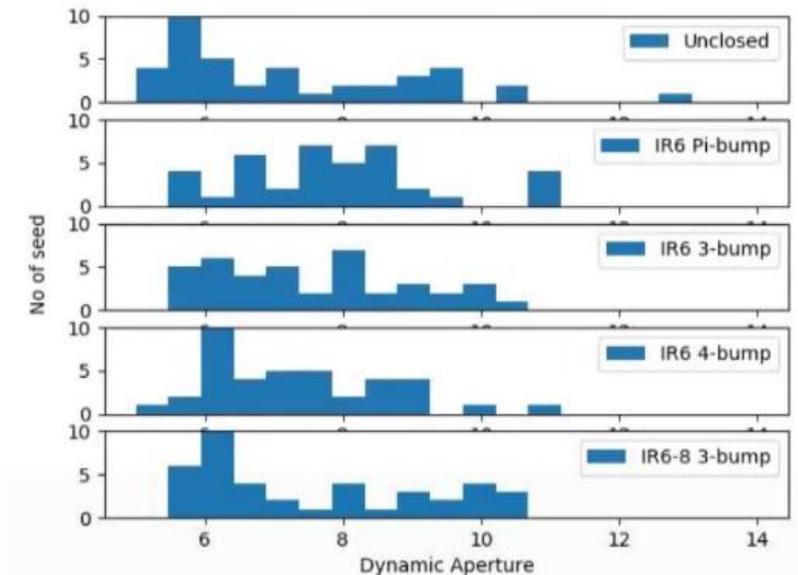
Unclosed crab dispersion (dx/dz) bump in the HSR lattice design



A: reference, B: w/o voltage adjustment, C: w/ voltage adjustment

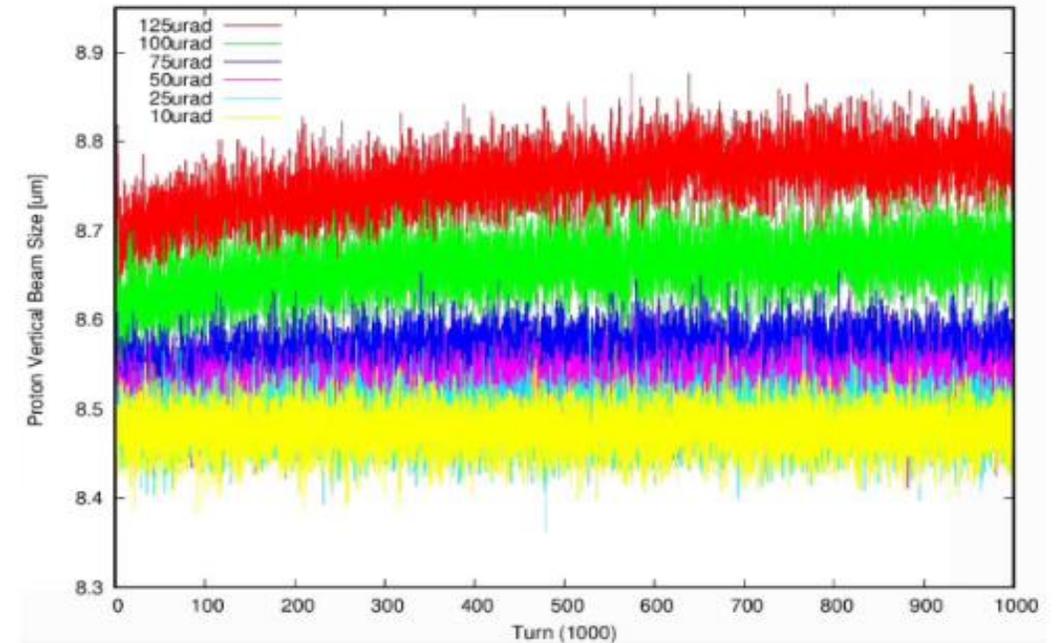


Histogram of DAs with 40 seeds of IR field errors



Vertical Crab Dispersion at IP in HSR

- Vertical crab dispersion (dy/dz) can be introduced by horizontal crabbing and betatron coupling (solenoids, skew quadrupoles) in the IR.
- Weak-strong beam-beam and dynamic aperture simulation studies show that **the tolerance of vertical crab dispersion at IP is ~ 20 urad in the HSR**, which requires effective correction.
- A solution has been found for ESR with local skew quadrupoles and skew windings on quadrupoles. The compensation scheme for the HSR is on-going.

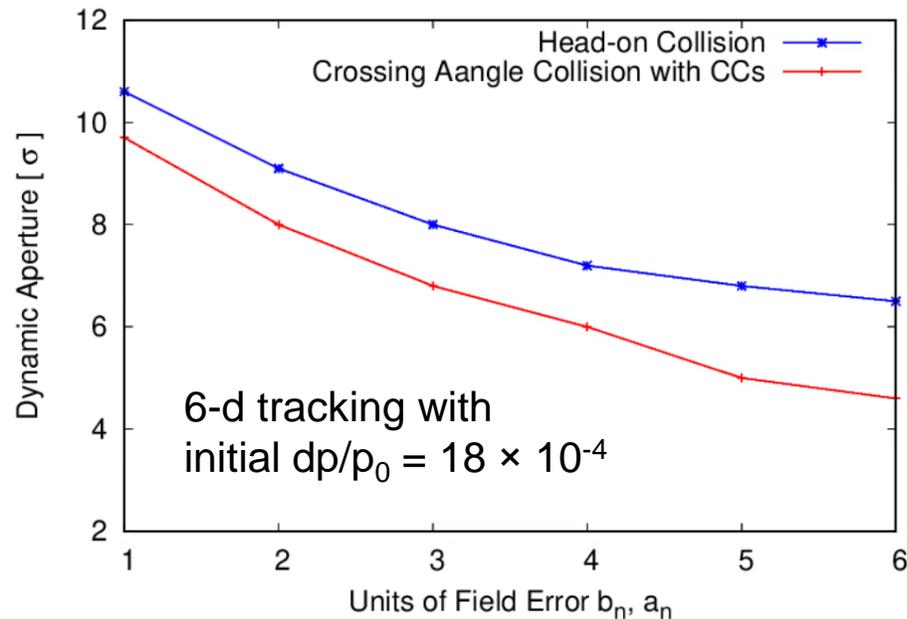


W-S simulation with different vertical crabbing

Magnetic Field Errors

- Based on RHIC's experience, IR magnetic field errors play an important role in DA reduction. To have a sufficient beam lifetime, DA with IR field errors and BB should be larger than 5σ .
- Magnetic field errors:

$$\Delta B_y + i\Delta B_x = B(R_{\text{ref}}) \left[10^{-4} \sum_{n=0}^{N_{\text{max}}} (b_n + ia_n) \frac{(x + iy)^n}{R_{\text{ref}}^n} \right]$$

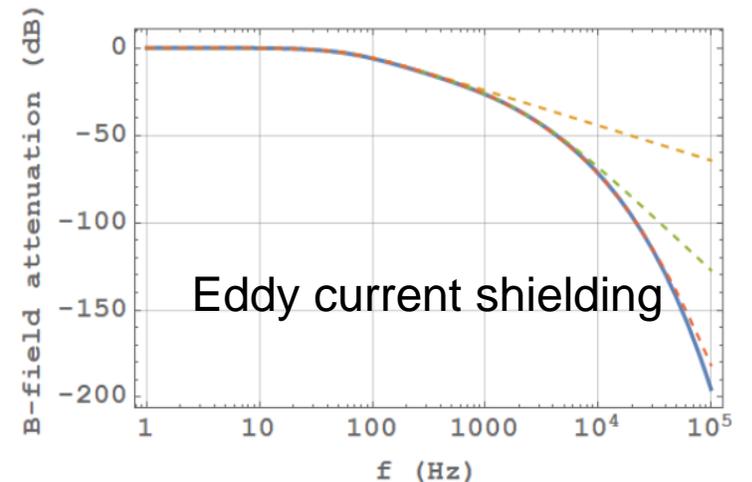
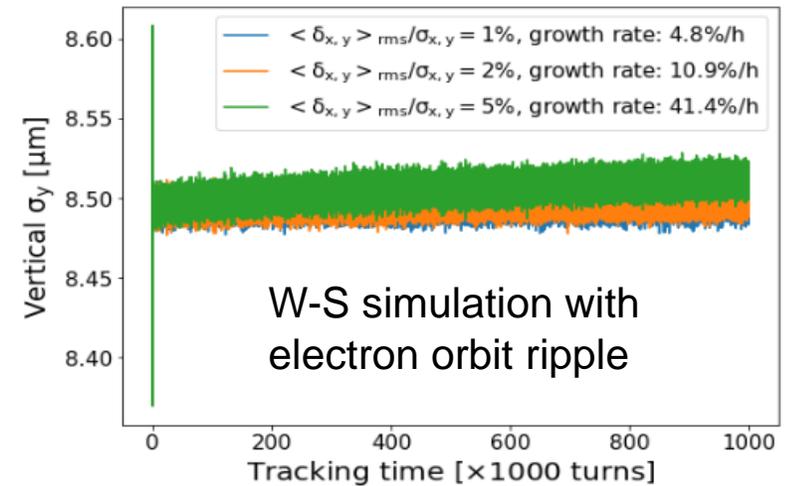


- Preliminary HSR DA results show that HSR DA is more than 6σ even with 3 unit of IR field errors, which is sufficient for beam lifetime.

- **We continue working on field error tolerances for HSR IR magnets**, especially for these large aperture magnets (e.g. , B0PF). We are in a close collaboration with the EIC magnet design team.

Power Supply Current Ripples

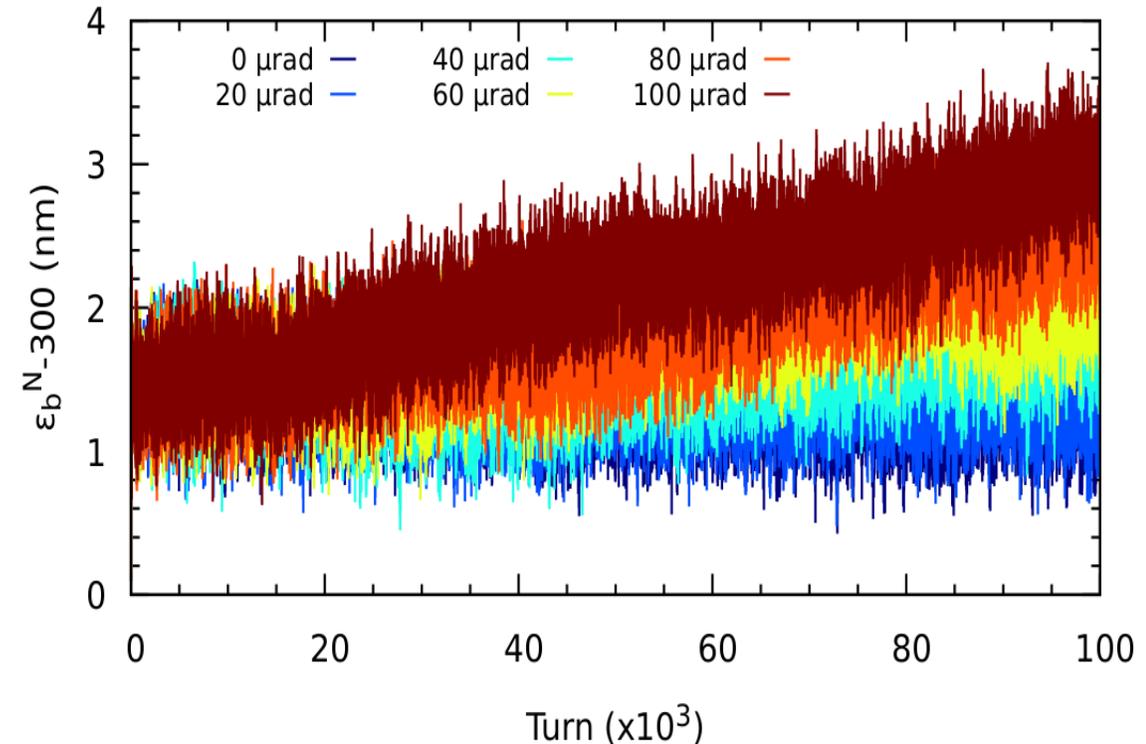
- Power supply current ripples, especially that from main dipoles of the ESR will introduce orbit oscillations, which will cause a sizeable proton emittance growth through BB.
- W-S BB simulation for highest luminosity collision mode: with proton beam size growth less than 10%/h, orbit oscillation at IP should be less than 2.5% $\sigma_{x,y}$ for low frequency band (<8kHz), and less than $10^{-4} \sigma_{x,y}$ for high frequency band.
- **The tolerance of dipole power supply current ripple at low frequency band is ~ 1 ppm.** The high-frequency ripple is less worrisome due to very significant eddy current shielding.
- Solutions under investigation: fast IP orbit feedback, grouping all dipoles on a same power supply, AC-couple all dipole PS, increase induction of ESR dipole PS, etc.



Crab Cavity Phase Noises

- Numerical simulations confirmed horizontal growth rates predicted by analytical calculation.
- Vertical emittance growth is observed when including beam-beam.
- To have proton beam size growth rate less than 10%/hour in both planes, **RMS of pink phase noises should be no more than 1 μrad** , which is beyond state-of-the-art.
- Countermeasures under investigation: RF phase feedback, fast one-turn beam feedback, high precision pickup 1 μm , etc. We are closely collaborating with CERN experts on this topic.

Vertical emittance evolution with phase noise and beam-beam (Bmad)



Summary

- In the talk we briefly reviewed the basics of beam-beam interaction and beam-beam effects in colliders.
- We heard the ongoing Electron-Ion Collider (EIC) project at Brookhaven National Laboratory.
- We went through major challenges of beam-beam interaction in the EIC design.

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